L.C. LEBBE (*)

MATHEMATICAL MODEL OF THE EVOLUTION OF THE FRESH WATER LENS UNDER THE DUNES AND BEACH WITH SEMI-DIURNAL TIDES

SUMMARY

The mathematical model of solute transport and dispersion of KONIKOW & BREDEHOEFT (1978) has been modified so that density-difference effects can be taken into account. With this two-dimensional model the evolution of the fresh-water lens under the dunes and under the shore was studied in a vertical cross-section. At initial time the whole aquifer is supposed to be filled with salt water. A constant infiltration rate of fresh water in the dunes and a constant hydraulic head under the dunes in function of the semi-diurnal tides is assumed. The vertical boundary under the sea is a constant-hydraulic-head boundary; the one under the dunes is a no-horizontal-flow boundary. The results are compared with field measurements. The hydraulic parameters which have an influence on the distribution of salts will be treated. Finally the effect of water withdrawal in the dunes on the salt distribution can be estimated by the mathematical model.

1. INTRODUCTION

In a former study the subterranean flow of fresh and salt water underneath the Western Belgian beach has been described [4]. For this study thirty borings were drilled on the gently sloping runnel and ridge beach through the

(*) Research Associate of the National Fund Scientific Research, Geological Institute, State University of Ghent, Belgium.

unconfined aquifer (Fig. 1). In each of the boreholes a resistivity logging was performed. Five resistivity profiles perpendicular to the shore line were drawn (Fig. 2). These profiles provide a fairly good idea of fresh, brackish and salt water distribution underneath the beach. At one of these profiles the hydraulic-head pattern has been measured continuously in the upper and the lower part of the aquifer. From these piezometers groundwater has been sampled for chemical analysis. Based on these data a mathematical model was developed. This two-dimensional mathematical model treated the steady-state flow of fresh and salt water with a sharp interface. It took also the density-difference effect into account. Thus the most important feature of the different fresh-salt water distribution can be explained.

Several examples were treated of different amounts of fresh-water flow from the dunes towards the sea. The results are represented in Fig. 3 where

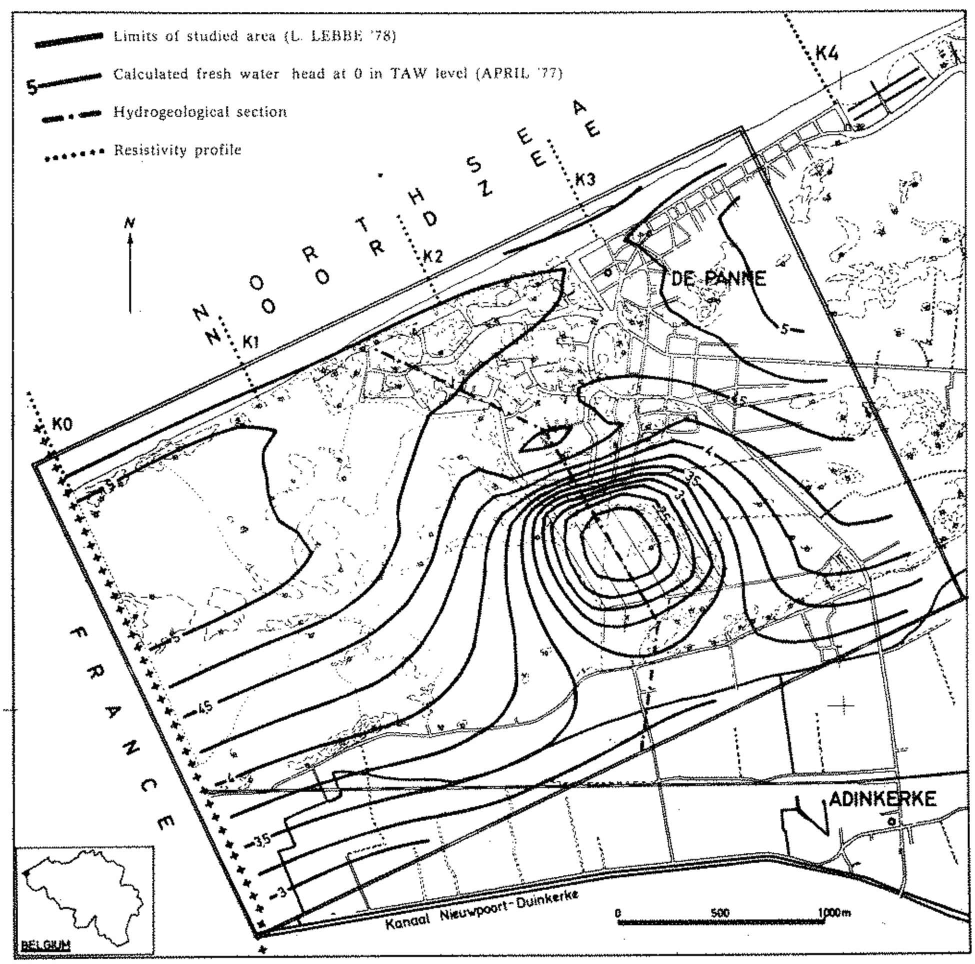

Fig. 1 – Situation of the hydrogeological study in the Westhoek area with calculated lines of equal hydraulic head and of the resistivity logging profiles.

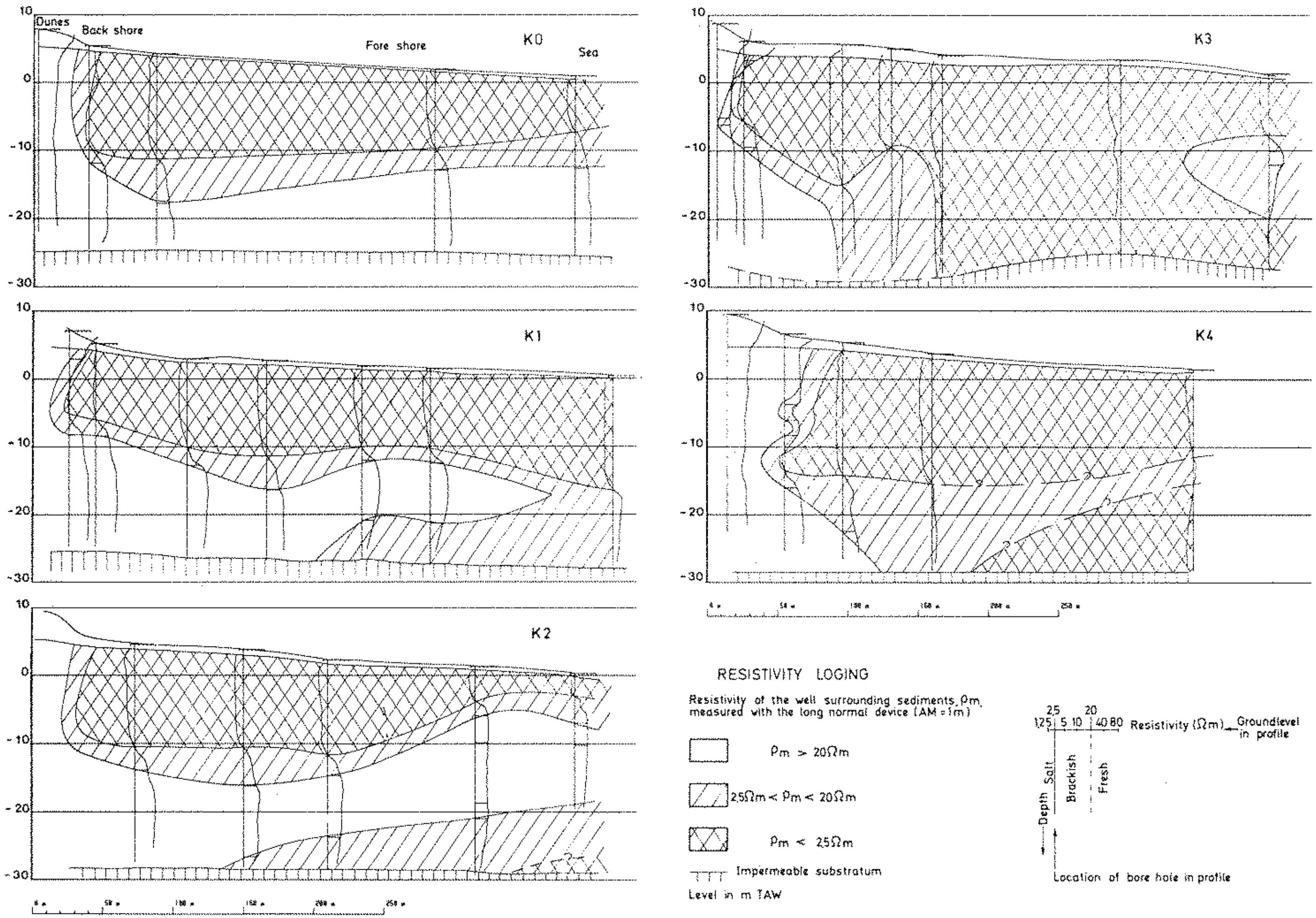

Fig. 2 – Resistivity profiles perpendicular to the shore line of the Westhoek area.

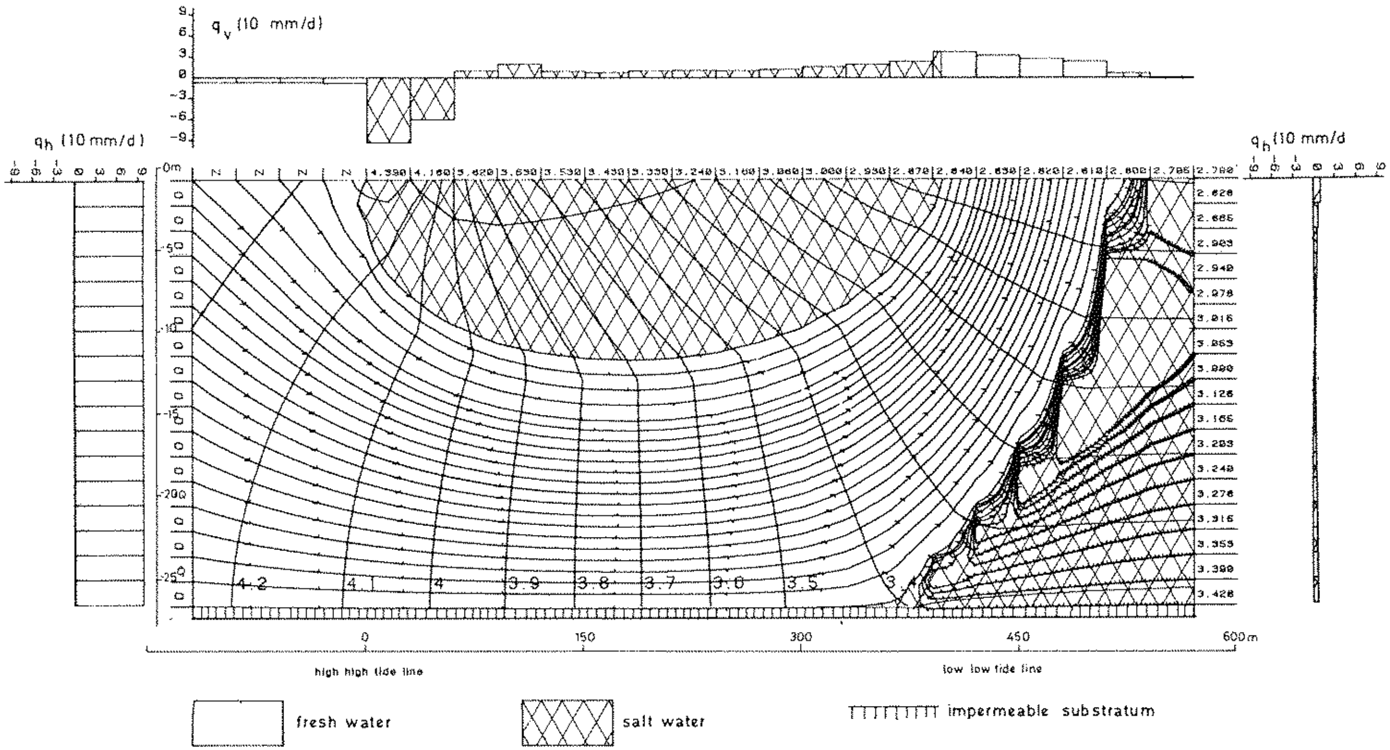

Fig. 3 – Flow of salt and fresh water and the lines of equal fresh-water head with a seaward fresh-water flow of $0.25 \cdot m^2/d$.

the lines of equal fresh-water head are shown together with the streamlines in the vertical plane of the two-dimensional model. The constant-flow-boundary conditions are indicated by the letter Q for the horizontal fresh-water flow of the dunes in the direction of the sea and the letter N for the vertical fresh-water flow, namely the infiltration rate in the dune area. The constant-head-boundary conditions are represented by the value of the constant fresh-water head. The inflow and outflow through the permeable boundaries of the mathematically treated area are calculated and represented near these boundaries. The streamlines have been drawn in such a way that they begin at a cell boundary and at a permeable boundary where an inflow of fresh or salt water occurs. The arrows on the stream lines indicated the end of the even years a water particle had travelled from the boundary of the area considered. For the last calculation a water-conducting porosity of 0.30 is taken into account.

A first example was given with a considerable seaward fresh-water flow (Fig. 3) Because of this flow a horizontal hydraulic gradient towards the sea underneath the dune area exists. Underneath the back shore and the upper part of the fore shore salt water infiltrates. It flows upon the fresh dune water. On the larger part of the fore shore the salt-water flow is directed vertically upwards resulting in an outflow at the beach surface. The fresh water only flows out at the bottom of the sea.

When the horizontal fresh-water flow through the vertical boundary is reduced the infiltration of salt water on the back shore and upper part of the fore shore is enlarged. Consequently the upper salt-water lens is enlarged in depth as well as in width. The fresh-water outflow is further offshore on the seabottom. The zone of fresh-water flow becomes narrower. The lower salt-water tongue retreats towards the sea.

In the last example treated (Fig. 4) the fresh-water flow through the vertical boundary under the dunes is reversed. Nearly all the infiltrating fresh water in the dune area is flowing towards this boundary. Only a small part of the infiltrating fresh water still flows towards the sea. On the back shore and upper part of the fore shore the infiltration of salt water becomes so large that the salt-water lens fills the whole aquifer under the shore. In spite of the very small fresh-water flow towards the sea there still is a tongue of fresh-water underneath the back shore.

In this mathematical model the process of hydrodynamic dispersion was not incorporated. It was also not possible to explain how much time it takes to evolve from one steady state to another and which intermediate states can occur. Therefore the mathematical model of solute transport and dispersion of Konikow & Bredehoeft [3] has been applied to this problem. Since in this mathematical model the hydraulic-head gradients were the only significant driving mechanism for the fluid flow it was first necessary to incorporate the effect of fluid density on the velocity distribution.

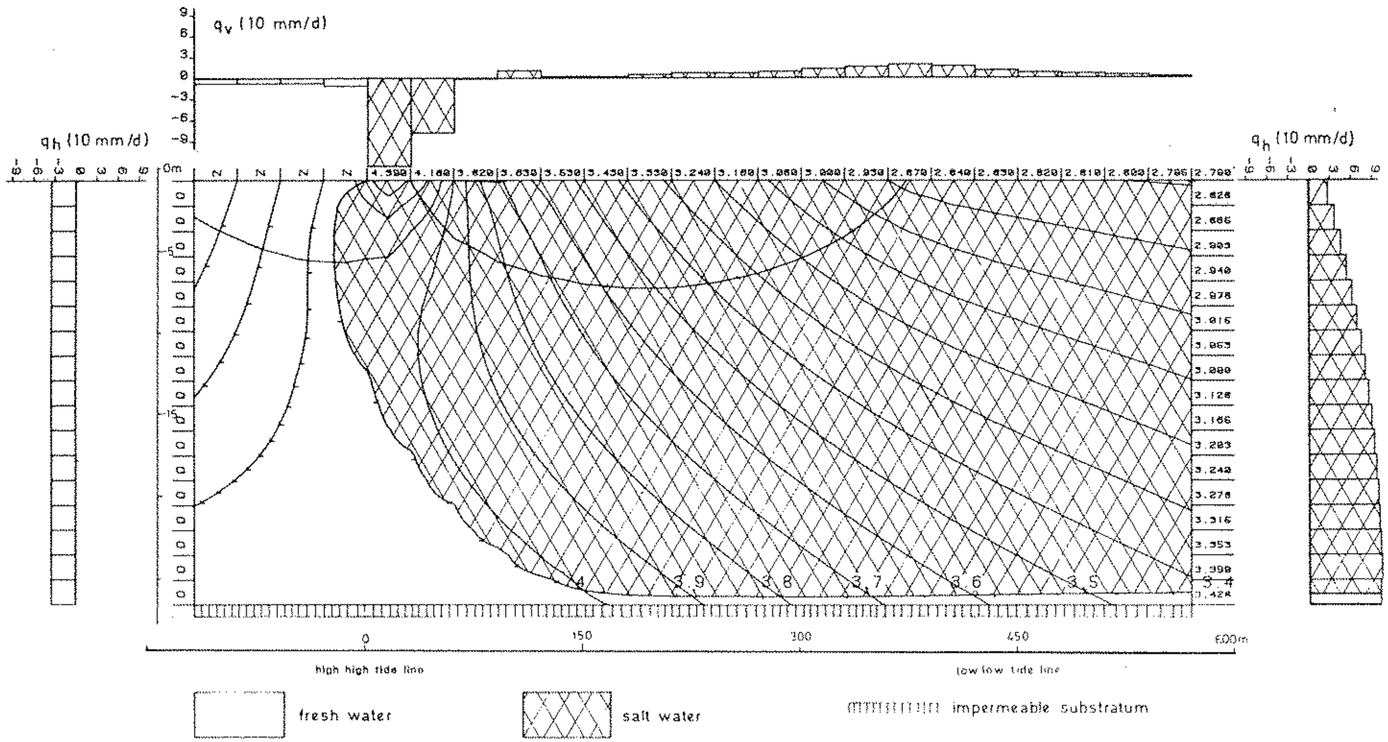

Fig. 4 - Flow of salt and fresh water and the lines of equal fresh-water head with a landward fresh-water flow of 0.09 m^2/d.

2. PROFILES OF FRESH-WATER PERCENTAGE

The five resistivity profiles enable us also to draw profiles of fresh-water percentage. This percentage of fresh water can be calculated if the salinity of the water in a point i of the groundwater reservoir is known together with the salinities of the fresh and salt water.

$$P_{fi} = 100\ F_{fi} = 100\ \left(\frac{c_s - c_i}{c_s - c_f}\right) \qquad (1)$$

where P_{fi} is the fresh-water percentage in a point i
F_{fi} is the fresh-water fraction in a point i
c_i is the salinity of the water in a point i
c_s is the salinity of the salt water
c_f is the salinity of the fresh water.

The salinity can be measured as the total dissolved solids, the chloride or electrical conductivity if we accept the simple linear relation between this parameters [5]. Considering the relation between the total dissolved solids and the resistivity of the water and a formation factor of 2.7 one obtains a relation between the resistivity of a sediment ρ_t, and the total dissolved solids TDS of the pore water.

$$\rho_{ti} = 32.400/TDS_i \qquad (2)$$

where ρ_{ti} is the resistivity of the sediments at a point i in Ωm
TDS_i is the total dissolved solids of the pore water at that point i in mg/1.

The total dissolved solids of native dune water (fresh water) is 450 mg/1 and of sea water (salt water) is 34.500 mg/1.

The resistivity of sediments filled by fresh-water following equation (2) is 72 Ωm.

The resistivity of sediments filled by salt water equals 0.95 Ωm. Combining equation (1) and (2) one finds the relation between the fresh-water percentage P_{fi} at a point i and the resistivity of the sediments ρ_{ti} at this point.

$$P_{fi} = 100 \cdot \left(\frac{\rho_{ti} \cdot \rho_{tf} - \rho_{tf} \cdot \rho_{ts}}{\rho_{ti} \cdot \rho_{tf} - \rho_{ti} \cdot \rho_{ts}}\right) \qquad (3)$$

The salt-water percentage is the complement of the fresh-water percentage.

$$P_{si} = 100 - P_{fi} \qquad (4)$$

The salt-water percentage can also be expressed in terms of salinities.

$$P_{si} = 100\ \left(\frac{c_i - c_f}{c_s - c_f}\right) \qquad (5)$$

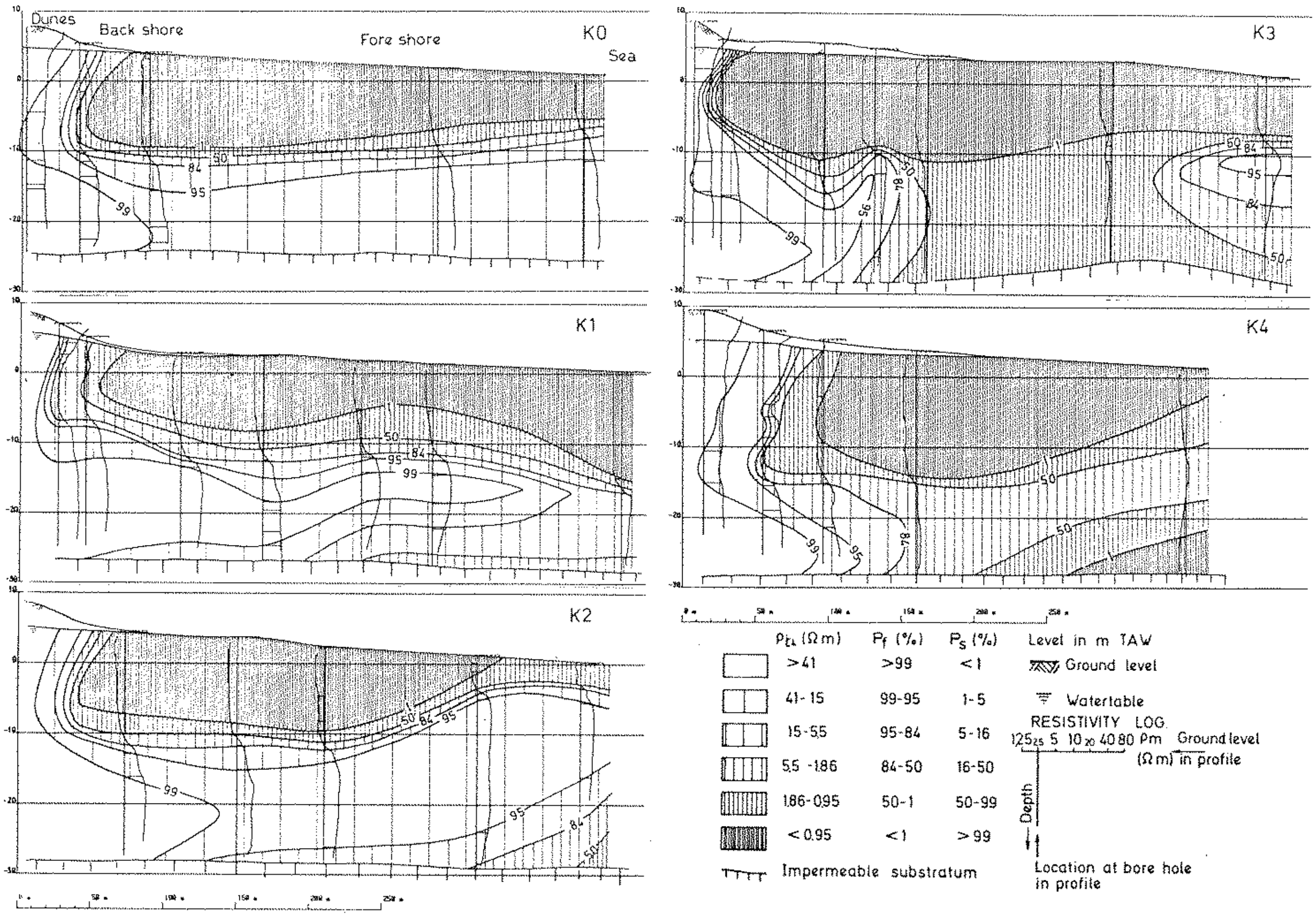

Fig. 5 – Profiles of fresh-water percentage.

This salt-water percentage is also called the relative salinity S_R [6].
Combining equation (2) and (5) results in a relation between the salt-water percentage P_{si} and the resistivity of the sediments ρ_{ti}.

$$P_{si} = 100 \cdot \left(\frac{\rho_{tf} \cdot \rho_{ts} - \rho_{ti} \cdot \rho_{ts}}{\rho_{ti} \cdot \rho_{tf} - \rho_{ti} \cdot \rho_{ts}}\right) \tag{6}$$

It is clear that the resistivity varies significantly when a small amount of salt water is mixed with fresh water (Fig. 5). The contrary is not true. As a consequence one can easily detect differences in admixtures of small amounts of salt water in fresh water. It is more difficult to detect differences in admixtures of small amounts of fresh water in salt water. Consequently the lines of equal fresh-water percentage are accurately drawn for large values. For smaller values of fresh-water percentage these lines cannot be drawn accurately.
The profiles of fresh-water percentage allow one to compare the field observations with the results of the mathematical model of solute transport and dispersion.

3. MATHEMATICAL MODEL

3.1. Theoretical background and numerical technique

The mathematical model of solute transport and dispersion in groundwater of Konikow & Bredehoeft [3] has been applied. This model calculates the transient changes in concentration of a non-reactive solute in flowing groundwater. The computer program solves simultaneously two partial differential equations. One equation is the groundwater flow equation, which describes the head distribution in the aquifer. The second is the solute-transport equation, which describes the chemical concentration in the system.
The computer program is modified so that density-difference effects can be taken into account. For this purpose the groundwater flow equation has been adjusted. The horizontal and vertical Darcian velocity in a point i of the groundwater reservoir was deduced from the equations (1) and (2).

$$q_{hi} = k_{hi} \frac{\rho_f}{\rho_i} \frac{\partial \psi_{if}}{\partial x} \tag{7}$$

$$q_{vi} = k_{vi} \frac{\rho_f}{\rho_i} \left(\frac{\partial \psi_{if}}{\partial z} + \frac{\rho_i - \rho_f}{\rho_f}\right) \tag{8}$$

where q_{hi} is the horizontal Darcian velocity (LT^{-1})
k_{hi} the horizontal hydraulic conductivity (LT^{-1})
ρ_f the density of the water (ML^{-3})
ρ_i the density of water at point i (ML^{-3})
$\partial\psi_{if}/\partial x$ the horizontal fresh-water head gradient (dimensionless)
q_{vi} the vertical Darcian velocity (LT^{-1})
k_{vi} the vertical hydraulic conductivity (LT^{-1})
$\partial\psi_{if}/\partial z$ the vertical fresh-water head gradient (dimensionless)
$(\rho_i-\rho_f)/\rho_f$ the buoyancy at point i (dimensionless).

The buoyancy is deduced from the chemical concentration or salinity which is the result of the solute-transport equation. A simple linear relation is assumed between the salinity at a point i and the density at this point. The contribution of higher degree terms in this relation can be ignored as well as the influence of the temperature and the pressure on this relation. Of course this is within the limiting values of salinity, temperature and pressure which can occur in the studied area.

The density of the fresh water having a salinity c_f (TDS = 450 mg/1) is taken as ρ_f(= 1 g/cm^3) and the density of salt water having a salinity c_s (TDS = 34,500 mg/1) is taken as ρ_s(= 1.025 g/cm^3). The linear relation between a density ρ_i and a salinity c_i and passing through the points ρ_f, c_f and ρ_s, c_s can be expressed as:

$$\rho_i = (c_i \cdot (\rho_s - \rho_f) + \rho_f c_s - \rho_s c_f) / (c_s - c_f) \quad (9)$$

The buoyancy can be expressed as a function of the salinity by means of equation (9):

$$(\rho_i - \rho_f) / \rho_f = (c_i - c_f) \cdot (\rho_s - \rho_f) / (c_s - c_f) \quad (10)$$

Substitution of equation (5) into equation (10) results in a simple relation between the buoyancy and the salt-water percentage or the relative salinity.

$$(\rho_i - \rho_f) / \rho_f = P_{si} \cdot (\rho_s - \rho_f) / (\rho_f \cdot 100) \quad (11)$$

The digital computer program uses an alternating-direction implicit procedure to solve a finite-difference approximation of the groundwater flow equation and it uses the method of characteristics to solve the solute-transport equation. The latter uses a particle-tracking procedure to represent convective transport and a two-step explicit procedure to solve a finite-difference equation that describes the effect of hydrodynamic dispersion, fluid sources and sinks and divergence of velocity.

3.2. Initial and boundary conditions

At the initial time an unconfined equifer is supposed to be completely filled with salt water. During the simulation period we suppose that the boundary conditions do not change.

The aquifer is bounded below by an impermeable layer. The landward vertical boundary is assumed to be the water-divide line or in the mathematical model a no-horizontal-flow boundary. The upper boundary of the aquifer is partially located under the dunes and partially under the beach and the sea. Under the dunes we assume a constant-vertical-flow boundary. This vertical flow rate is equal to the infiltration rate of fresh water in the dune area. Beneath the beach and the sea a constant hydraulic head boundary is assumed. The values of these hydraulic heads were deduced from measurements on the beach [4]. Where a downward vertical flow occurs on the beach there is an infiltration of salt water. The seaward vertical boundary is a constant-hydraulic-head boundary. The salt-water head is the same over the whole depth. The density or salinity of the water flowing through this boundary depends on the calculated flow direction. When the flow is inward the studied area salt-water enters.

3.3. Sequences of calculations

Because we assume at the initial time an unconfined aquifer completely filled with salt water the buoyancies between the nodal points are equal. With the given boundary conditions and the known buoyancies between the nodal points the fresh-water head at each nodal point is derived for the first time step. With the fresh-water-head differences and the buoyancies between the nodal points the velocity components between these nodal points are calculated by the equations (7) and (8). By using these components the solute transport equation is solved by applying the method of characteristics. From these results the chemical concentrations or salinities are deduced between the nodal point at the end of the first time step. With this last results the buoyancies between the nodal points are calculated ofter equation (11). They are used for the calculation of the fresh-water heads and velocity components for the second time period. By the method of characteristics the salinity is calculated after the second time step, where after the buoyancies can again be calculated for the next time step.

3.4. Input

In the two-dimensional model the aquifer is considered in a vertical plane. The aquifer is subdivided in ten layers with a thickness of three meters and thirty-eight columns with a width of thirty meters. Eighteen columns are located under the dunes, fourteen under the beach and six under the sea. In this case the water-divide was supposed to be at 540 m from the high-high-water line and the low-low-water line at 420 m from the high-high-water line. The offshore is considered over a widthof 180 m. The mean inflitration rate of fresh water under the dunes is $7.39.10^{-4}$ m/d. The horizontal hydraulic conductivity is held constant over the considered aquifer at a value of 10 m/d, the vertical hydraulic conductivity is also held constant at a value of 0.2 m/d. By the application of the method of characteristics nine traceable particles, where a concentration is assigned, are used in each cell of the finite-difference grid. The maximum cell distance per movement of the particles has been chosen at 0.85. The longitudinal dispersivity of the porous medium, α_L, is 0.15 m. The ratio of the transverse dispersivity to the longitudinal dispersivity, α_T/α_L, is 0.3. The effective or water-conducting porosity, ε, is 0.3. After each time step of five years the buoyancy was deduced from the calculated concentration and a new fresh-water head and groundwater velocity configuration was recalculated.

First the evolution of the fresh-water lens under the dunes and beach was simulated during a period of five hundred years. Finally the effect of fresh-water withdrawal in the dunes was simulated. In the eighteen nodal points indicated in figure there was a water withdrawal. During the first five years the total amount of water pumped was 0.10 m^2/d. The second five years this amount becomes 0.21 m^2/d and finally stays equal at 0.42 m^2/d during a period of thirthy years. This last total amount of water pumped exceeds the total amount of infiltration of fressh-water in the dune area i.e. 0.40 m^2/d.

3.5. Output

For every nodal point the fresh-water head and the groundwater flow velocities are obtained for every time step, the chemical concentration after every time step. This output data of the different time steps can be represented in a vertical crossections through the aquifer. In this cross-section the vertical axis is exaggerated with respect to the horizontal axis. The fresh-water heads during the time step are represented by lines of equal fresh-water head. These lines are found by a linear interpolation between the values of the nodal points. The groundwater flow velocities during the time step are represented by vectors. These vectors are obtained by calculating the horizontal and vertical components in the nodal points. These components are found by the multiplication of the horizontal or vertical velocities with a time increment. This time increment was chosen in fig. 6 and fig. 7 as one half year. The vectors are plotted in all nodal points taking the units of the horizontal and vertical axis into account. The concentration at the end of every time step is represented by lines of equal percentage of fresh water. These lines are again obtained by a linear interpolation between the concentration of the nodal points.

3.6. Discussion of results

With the two-dimensional model which treats the steady state flow of fresh and salt water with a sharp interface and takes the density difference into account only the stable fresh-salt-water distribution could be explained. Such a stable salt-fresh-water distribution occurs at the Belgian-French border.

There the natural groundwater flow towards the sea is the least affected. It was already more difficult to explain the fresh-salt-water distribution before the urban area of De Panne. This was in particular the case for the occurrence of the isolated lens of brackish water under the lower part of the fore shore (K3 and K4 in Fig. 2).

By the application of the two-dimensional model of solute transport and dispersion which takes density difference into account one becomes intermediate states in the evolution from one steady state to another. Looking to the results of our first run represented in Fig. 6 and Fig. 7 we can already explain more feature of water-quality distribution.

For obtaining the water-quality distribution under dunes, beach and sea one has first simulated the evolution for a sufficient long period until it changes no longer meaningfull. At the initial time the aquifer was supposed to be filled with salt water. From this time point on the sea did not inundate the young dune ridge and the water starts to infiltrate through the unsaturated zone.

The salt water is driven out and a fresh-water lens starts to form. During the first years a large transition zone exists between this fresh-water lens and the salt water. Already at the beginning of the formation of the fresh-water lens a brackish-water tongue starts to form in the upper part of the aquifer. In course of time the fresh-water lens grows and the brackish tongue becomes larger, less mineralized and sinks. The washout of the last salt particles near the impermeable substratum occurs first and the most under the back shore and last and the least under the water-divide in the dunes. Finally the brackish tongue becomes fresh although the washout is very slowly.

Under the sea the salt-water flow is considerable principally horizontal and in the seaward direction during the first years. When time goes on the salt water becomes a less mineralized and the flow diminishes continuously. In the lower part of the aquifer this flow inverses and the water becomes again more mineralized. In the upper part of the aquifer this flow stays in the seaward direction. A very slow flow of salt water develops from the lower to the upper part of the aquifer as was already discribed by Cooper et al [1].

After five hundred years of simulation the hydraulic-head and water-quality distribution under dunes, beach and sea does not further change meaningfull. This hydraulic-head and water-quality distribution are now used as initial values for the simulation of the evolution of the water-quality distribution during pumpage in the dunes. After ten years of pumpage the total pumpage, Q_{tot}, is held at 105% of the total infiltration of fresh-water, N_{tot}^{fresh}. During the first five years the total pumpage is a quater of the total pumpage after ten years while it is a half during the second five years. This simulation is run to study the effect of over-pumpage of fresh-water in the dunes.

From the moment on that the pumpage starts the salt-water lens in the upper part under the shore extends. This happens especially under the fore shore. The fresh-water tongue under the beach becomes narrower and more mineralized. When over-pumpage starts two important developments take place in the water-quality distribution. The first take place under the boundary of dunes and beach, the second under the fore shore and the sea. Under the boundary of the dunes and beach salt water starts to flow landwards. A vertical transition zone develops. When over-pumpage goes on the developing salt-water encroachment under the dunes sinks until it reaches the lower part

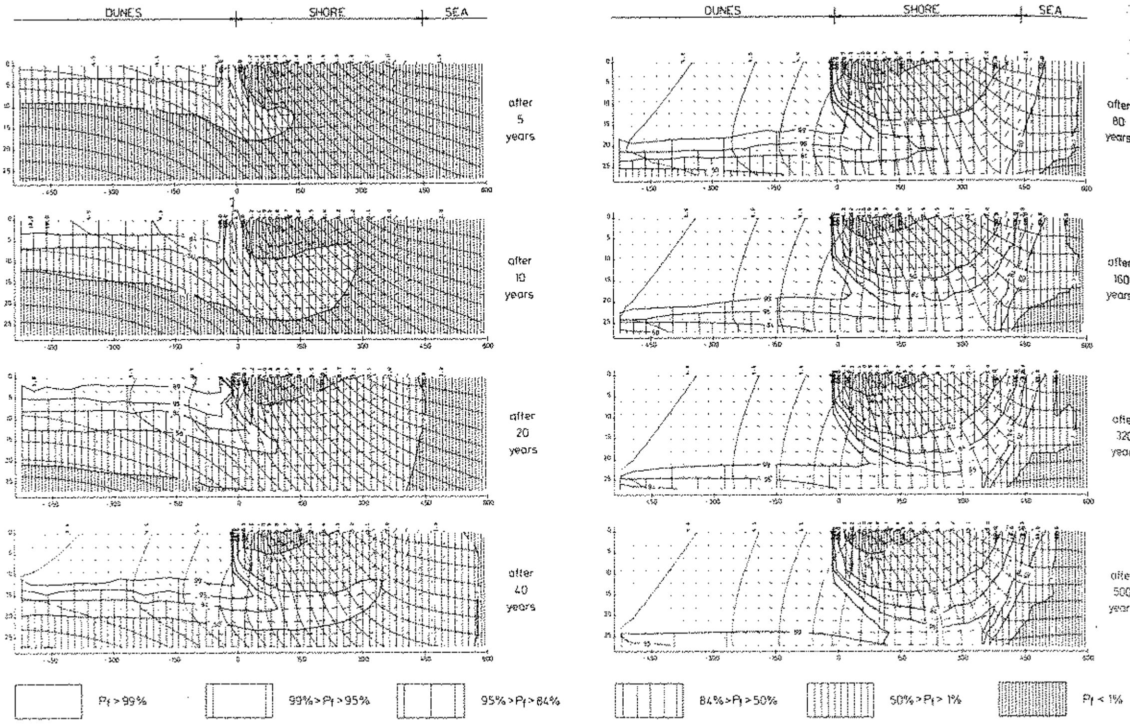

Fig. 6 – Evolution of fresh-water lens under dunes, beach and sea.

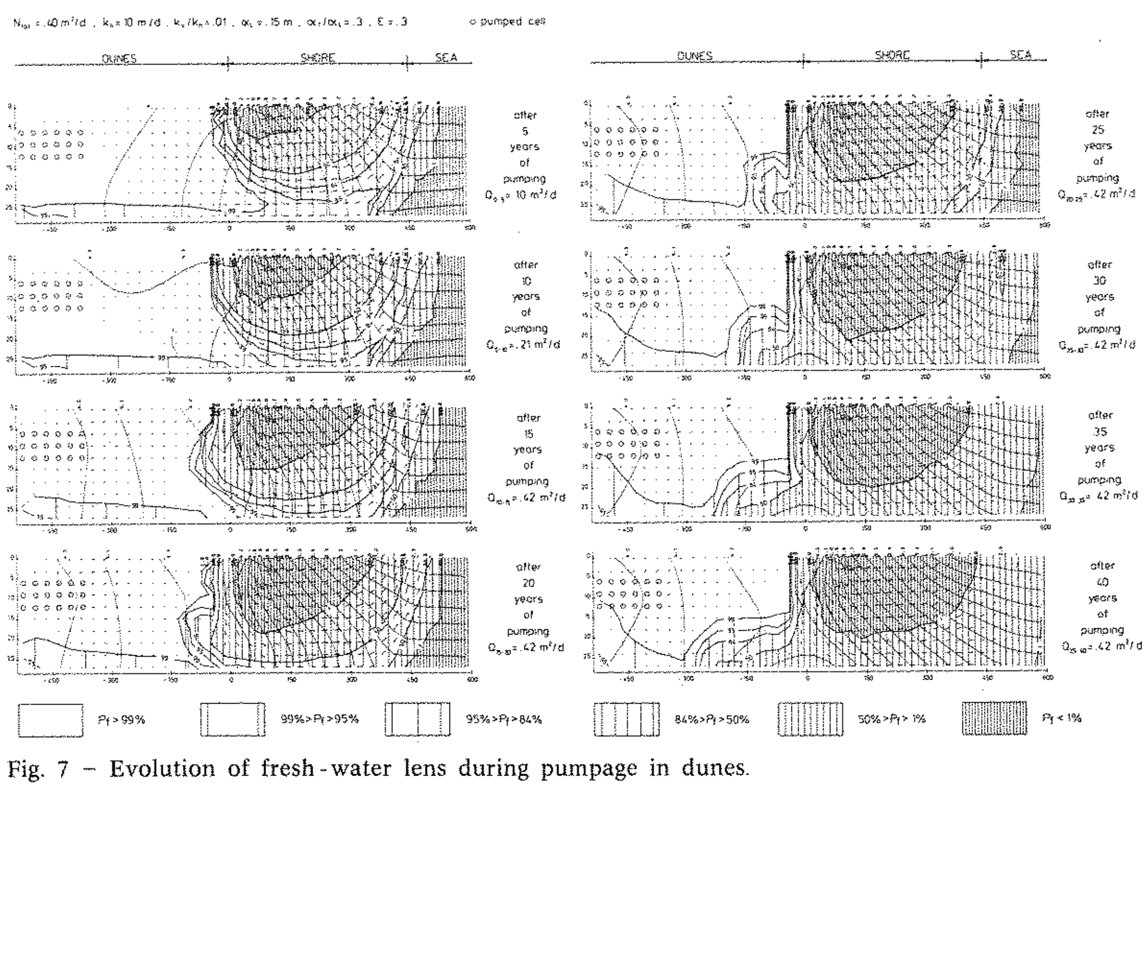

Fig. 7 – Evolution of fresh-water lens during pumpage in dunes.

ot the aquifer. Finally the salt-water encroachment advances in the lower part of the aquifer. The fresh-water which is a less more mineralized rises from the proximity of the impermeable substratun to the water-catchment. Under the fore shore and the sea the fresh-water tongue become more mineralized and the salt-water lens sinks especially under the fore shore so that an isolated brackish lens is squeezed of the tongue. When time went on this brackish lens become more and more mineralized and moves upwards and in the direction of the sea. This explains the existence of the brackish lenses under the lower part of the offshore before the urban area of De Panne.

From the above given description it is clear that models gains an insight into the evolution of salt-water encroachment and can help to take measure against this danger. Nevertheless these models can be applied, field data stay indispensable to calibrate these. Only a good combination of accurate field data like hydraulic heads, water-conducting properties of the sediments and water-quality distribution with the models can help us attaining the above mentioned goals. As KONIKOW [2] mentioned the model does not replace field data, but it does offer a feedback mechanism that can help to guide the design of more effective and more efficient data collection programs.

Finally accurate hydrogeological field data and the use of models of solute transport and dispersion can help us to check some proposed geological and geomorphological evolution of some parts of the coastal plain. This help will rather be in the exclusion of some possibilities. A direct deduction of the evolution is rather difficult although not wholly excluded in some cases. For the simulation of the evolution of the water quality under the coastal plain during geological times a good knowledge of the evolution of the topography, and the type, amplitude and mean level of the tidal movement are necessary together with the estimates of the infiltration rates of fresh-water during this time.

The calibration of models of solute transport and disperion in groundwater is interesting because one can easily collect accurate data about the water-quality distribution by means of resistivity logging. This is in particular true for low salt-water percentages (or low relative salinities).

The above described simulation was only a first run. The calibration of the model should be prolonged. By sensitivity analysis we get insight in the effect of the change of the different parameters. The calibration must first take place on the measured hydraulic heads and later on the water-quality distribution. So we hope to obtain more insight in the solute transport and dispersion processes.

4. CONCLUSIONS

Resistivity logging enables us to draw profiles of fresh-water percentage. When fresh-water percentage is high the resistivity varies significantly with change of fresh-water percentage. As a consequence it is easy to detect differences in mixture of small amounts of salt water in fresh water. In the profiles of fresh-water percentage the line of equal percentage of fresh-water are drawn for the values 1, 50, 84, 95, 99. In this way the field observation of water-quality can easily be compared with the results of the mathematical model of solute transport and dispersion.

The mathematical model of solute transport and dispersion of KONIKOW & BREDEHOEFT [3] has been modified so that density-difference effects can be taken into account. With this two-dimensional model the evolution of the fresh-water lens under dunes and beach was simulated during five hundred years. This results in a fresh-water head and water-quality distribution which have been used as initial values for the simulation of the evolution of the fresh-water lens under the dunes during pumping.

Field data such as hydraulic-head measurements and resistivity logging ables us to calibrate this model. The result of the model help us to plan collection of field data. The calibrated model on the field data can be a helpfull tool in the control of the fresh-water supply of the dunes of the western coastal plain. The collection of other field data such as porosity and hydraulic conductivities is advisable.

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