SIMPLE GROUNDWATER FLOW MODELS FOR SEAWATER INTRUSION

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ABSTRACT

A method for the simulation of seawater intrusion is presented based on three major approximations: the Dupuit and Boussinesq approximations and by neglecting diffusion and dispersion. The method is intended for the solution of practical problems of seawater intrusion in coastal aquifers and may be applied to interface flow and variable density flow in both two and three dimensions. The upconing of an interface in a vertical cross-section is presented as an example. The solution approaches the steady-state solution in 500 years.

INTRODUCTION

The objective of this paper is to present a simple approach for the modeling of seawater intrusion in coastal aquifers. The approach is intended for the solution of engineering problems in coastal aquifers that have the following three characteristics.

1. Groundwater flow is essentially horizontal; gradients of instantaneous streamlines rarely exceed 10 percent.
2. The difference in density between freshwater and seawater is relatively small, on the order of 2.5 percent.
3. The time period of interest is relatively short, on the order of 100 years.

For such problems it is possible to simplify the set of mathematical equations that describes flow in the aquifer significantly by making the following three approximations (corresponding to the aforementioned three characteristics).

1. The resistance to flow in the vertical direction in neglected within an aquifer (the Dupuit approximation).
2. The mass balance equation for groundwater flow is replaced by the flow balance equation (the Boussinesq approximation); density effects are taken into account through Darcy's law.
3. Freshwater and seawater are treated as two immiscible fluids (in the case of interface flow), or diffusion and dispersion are neglected and brackish water and saltwater move through the aquifer by advection only (in the case of variable density flow).

In addition to these three major approximations, a number of minor, common approximations are adopted, such as: the viscosity of the freshwater and saltwater are equal, and the aquifer is piecewise homogeneous and isotropic.
A model based on these three approximations may be formulated in terms of verti-

cally integrated fluxes and does not require a vertical discretization of the aquifer. It does need a three-dimensional definition of either the shape of the interface or the density distribution, however. The formulation is especially suited for the implementation in existing (numerical) models for single density groundwater flow.

BACKGROUND

Dietz (1953) was probably the first to describe interface flow in terms of integrated fluxes; he developed equations for the specific discharge vector. Essaid (1990) developed an interface formulation based on integrated flows without adopting the Boussinesq approximation and wrote the computer program SHARP. Variable density flow was formulated with integrated fluxes and expressed in pressures by Maas and Emke (1988). Strack (1995) used vertically integrated fluxes to derive a comprehensive potential for variable density flow; Bakker (1998) did the same for interface flow. The formulation presented here is written in terms of freshwater heads, which makes it suitable for the implementation in existing, numerical groundwater codes for single density flow.

INTERFACE FLOW IN A VERTICAL CROSS-SECTION

As an example, equations for one-dimensional interface flow in a single aquifer are derived. An *x,z* Cartesian coordinate system is adopted with the *z* axis pointing vertically upward. Darcy's law for the specific discharge in the *x* direction yields

$$q_x = -k\frac{\partial \phi}{\partial x}$$

where k is the hydraulic conductivity of freshwater and ϕ is the equivalent freshwater head. The Dupuit approximation is adopted, which means that the resistance to flow in the vertical direction is neglected within an aquifer and the pressure is hydrostatic within an aquifer. As a result, the freshwater head may be written as

$$\phi = h \qquad z \geq \zeta$$
$$\phi = h + \nu_s(\zeta - z) \qquad z \leq \zeta$$

where h is the head in the freshwater zone, ν_s is the dimensionless density difference [$\nu_s = (\rho_s - \rho_f)/\rho_f$], and ζ is the elevation of the interface. Combination of Darcy's law and the definition of the freshwater head gives

$$q_x = -k\frac{\partial h}{\partial x} \qquad z \geq \zeta$$
$$q_x = -k\frac{\partial h}{\partial x} - k\frac{\partial \zeta}{\partial x} \qquad z \leq \zeta$$

The vertically integrated flux, referred to as the discharge vector, may be obtained as

$$Q_x = \int_{z_b}^{z_t} q_x dz$$
$$= -k(z_t - z_b)\frac{\partial h}{\partial x} - k\nu_s(z_t - z_b)\frac{\partial \zeta}{\partial x}$$

where z_b and z_t are the elevations of the saturated bottom and top of the aquifer, respectively. The transmissivity, $T = k(z_t - z_b)$, and the newly introduced 'salt' transmissivity, $\tau = k\nu_s(z_t - z_b)$, may be used to write the discharge vector as

$$Q_x = -T\frac{\partial h}{\partial x} - \tau\frac{\partial \zeta}{\partial x}$$

The Boussinesq approximation is adopted, which means that the mass balance equation is replaced by a volume balance. As a result, the divergence of the discharge vector is equal to the total inflow

$$\frac{\partial Q_x}{\partial x} = N - S\frac{\partial h}{\partial t}$$

where N is the infiltration of freshwater and S is the storage coefficient, equal to the effective porosity for unconfined flow. Differentiation of the discharge vector finally gives

$$\frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right) = -N + S\frac{\partial h}{\partial t} - \frac{\partial}{\partial x}\left(\tau\frac{\partial \zeta}{\partial x}\right) \tag{1}$$

Notice that the presence of the interface results in one additional term, $-\partial(\tau\partial\zeta/\partial x)/\partial x$, on the right-hand side of the equation. This term must be added to the source term of a single-density groundwater model to take into account the effect of an interface.

MOVEMENT OF THE INTERFACE

An equation for the movement of the interface may be obtained by writing the continuity equation for flow in the saltwater zone. The vertically integrated flux in the saltwater zone, referred to as the 'salt' discharge, may be written as

$$\overset{s}{Q}_x = \int_{z_b}^{\zeta} q_x dz$$
$$= -k(\zeta - z_b)\frac{\partial h}{\partial x} - k\nu_s(\zeta - z_b)\frac{\partial \zeta}{\partial x}$$
$$= -\frac{\tau}{\nu_s}\frac{\partial h}{\partial x} - \tau\frac{\partial \zeta}{\partial x}$$

Continuity of flow in the saltwater zone gives for the divergence of the salt discharge

$$\frac{\partial \overset{s}{Q}_x}{\partial x} = -n\frac{\partial \zeta}{\partial t}$$

where n is the effective porosity. Differentiation of the salt discharge gives

$$\frac{\partial}{\partial x}\left(\tau\frac{\partial \zeta}{\partial x}\right) = n\frac{\partial \zeta}{\partial t} - \frac{\partial}{\partial x}\left(\frac{\tau}{\nu_s}\frac{\partial h}{\partial x}\right) \tag{2}$$

It may be observed from (2) that at steady state, the Ghyben-Herzberg condition will be fulfilled

$$\zeta = -\frac{h}{\nu_s}$$

and the interface does not move ($\partial\zeta/\partial t = 0$).

SOLUTION PROCEDURE

A solution is obtained through the consecutive solution of equations (1) and (2) with appropriate boundary conditions. An implicit, block-centered finite difference scheme is used (much like MODFLOW; McDonald and Harbaugh, 1988). Starting with an initial position of the head and interface at time t, the head distribution at time $t+\Delta t$ is calculated with (1), keeping the interface position fixed. The position of the interface at $t+\Delta t$ is obtained by solving (2) while keeping the head distribution fixed.

EXAMPLE

The upconing of an interface below an infinitely long ditch parallel to the coast is simulated as an example. A vertical cross-section normal to the coast is considered. Freshwater is extracted from the ditch, which extends into the freshwater zone only; flow is two-dimensional in the vertical plane. Initially, flow in the confined aquifer is uniform towards the coast and the interface is at the steady state position (thick line, Figure 1a). Starting at time zero, half the uniform flow is extracted from the ditch. The upconing of the interface is simulated.

Parameters

The following parameters were used: hydraulic conductivity k = 2 m/d, effective porosity n = 0.2, storage coefficient S = 0, thickness of the aquifer H = 45 m, bottom of the aquifer z_b = – 45 m, top of the aquifer z_t = 0 m, length of the cross section L = 1000 m, width of the cross-section normal to the plane of flow W = 1 m, dimensionless density ν_s = 0.025. The ditch is located at a distance of 590 m from the coastline at x = 0.

The boundary conditions were specified as follows. The uniform flow towards the coast is Q_0 = 0.1 m^2/d. The boundary condition at the coast is obtained with Glover's formula (1959)

$$\zeta_0 = \frac{-Q_0}{k\nu_s}$$

At x = 0 m the head in the freshwater zone is fixed at h_0 = 0.05 m and the interface at ζ_0 = – 2 m. Glover's solution was applied for the specification of a boundary condition at the coast by, among others, Person et al., and the effect was investigated by Bakker (2000a).

At t = 0, the discharge of the gallery is set to Q = – 0.05 m^2/d.

Results

The domain was discretized in 101 cells of length Δx = 10 m. A time step of Δt = 0.5 year was used. The position of the interface during upconing, with an interval of 50 years, is shown in Figure 1a. Although the method is intended for relatively short times, the simulation has been continued for 500 years to demonstrate that the interface approaches the steady state position (the dashed line).

It may be of interest to evaluate the influence of the boundary condition at the coastline. The head and interface are fixed at a position corresponding to an outflow of Q_0 = 0.1 m^2/d. When water is pumped from the ditch, however, the outflow is reduced to Q_0 = 0.05 m^2/d. The boundary condition corresponding to this outflow, obtained with Glover's formula, is h_0 = 0.025 m and ζ_0 = – 1 m. Strictly speaking, the boundary condition will change over time to these values. The influence of this change is insignificant (at least for this example) as is shown in Figure 1b, where the boundary condition is changed instantaneously at t = 0.

CONCLUSION AND DISCUSSION

A method was presented for the simulation of seawater intrusion based on three major approximations: the resistance to flow in the vertical direction is neglected within and aquifer (Dupuit approximation), the mass balance is replaced by the volume balance (Boussinesq approximation), and diffusion and dispersion are neglected. Equations were derived for interface flow in a vertical cross-section as an example. The resulting set of two differential equations may be implemented easily in existing groundwater models for single-density flow. An example was presented for the upconing of an interface below a ditch. The interface approaches the steady-state solution. The presented method may be applied in a similar fashion to solve both three-dimensional interface flow and variable density flow (Bakker, 1999, 2000b).

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Figure 1. Upconing of interface below a gallery. (a) 500 year simulation, time interval 50 years, initial position (thick), steady-state position (dashed); (b) 100 year simulation with original boundary condition (solid), and adjusted boundary condition (dashed)