SALT WATER MODELING AND SIMULATION OF LLOBREGAT DELTA AQUIFER

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ABSTRACT

In this paper a sharp interface numerical model is developed to simulate seawater intrusion into coastal aquifers. The model takes into account the flow dynamics of salt water and fresh water assuming a sharp interface between the two fluids. The cell-centred finite volume method is adopted here to solve the set of simultaneous partial differential equations describing the motion of salt water and fresh water separated by a sharp interface. These equations are based on the Dupuit approximation and are obtained from integration over the vertical dimension. In order to have flexibility upon complex configurations domain, non structured grid meshing is utilized. To approximate the diffusion fluxes, Green-Gauss type reconstruction, based on diamond-cell and least squares interpolation, is performed. The model is first validated using academic test case studies with known close form, and is applied to the case of salt water intrusion into the Llobregat delta aquifer, near Barcelona, Spain. The numerical and geophysical results are compared. Close agreements are achieved.

Key Words : Coastal aquifers, seawater intrusion, finite volumes method, unstructured meshes, Green-Gauss reconstruction.

INTRODUCTION

The relative good quality of groundwater, makes it to be the main source of fresh water supply. With the increase of population at alarming rates, the fresh water supply is being continually depleted increasing the importance of groundwater monitoring. One of the major concerns, most commonly encountered in coastal aquifers is caused by induced flow of salt water into fresh water aquifers, caused by excessive groundwater development currently known as salt water intrusion. The modelling of groundwater in coastal aquifers is an important and difficult issue in water resources. The primary difficulty resides in efficient and accurate simulation of the movement of the salt water front. Fresh water and salt water are miscible fluids and therefore the zone separating them takes the form of a transition zone caused by hydrodynamic dispersion. For certain problems where the transition zone is relatively small compared to the aquifer extent and thickness, the simulation can be simplified by assuming that the two fluids are immiscible and separated by a sharp interface. This last assumption, together with the Dupuit approximation, permits the integration of the equations in the vertical direction (Bear, 1979). The purpose of this paper is to present a cell-centered finite volume based approximation to calculate the position of the sharp interface. This class of methods is becoming one of the commonly used techniques for partial differential equations in engineering calculations and computational physics. Their popularity is mainly due to their ability to represent and reproduce the physics conservation and the possibility of solving the problems on complex geometries. The diffusive contributions here are discretized using Green-Gauss type interpolation. The gradient on each edge of a cell is approximated using the Green theorem combined to an interpolation. Numerical results obtained methods (geophysics and modeling) are compared.

MATHEMATICAL MODEL

Assuming that the salt water and fresh water are separated by a sharp interface, two domains are considered. For each flow domain the equation of continuity may be integrated over the vertical dimension reducing the determination of the position of the interface to a 2D problem.
The equations for fresh water and salt water flow, respectively, are written as follows (Essaid, 1990):

$$S_f\,B_f\,\frac{\partial h_f}{\partial t} + n\,\alpha\frac{\partial h_f}{\partial t} + \left[n\,\delta\frac{\partial h_f}{\partial t} - n\,(1+\delta)\frac{\partial h_s}{\partial t}\right] = \frac{\partial}{\partial x}\left(B_f\,K_{f_x}\frac{\partial h_f}{\partial x}\right) + \frac{\partial}{\partial y}\left(B_f\,K_{f_y}\frac{\partial h_f}{\partial y}\right) + Q_f \quad (1)$$

$$S_s B_s \frac{\partial h_s}{\partial t} + \left[n\,(1+\delta) \frac{\partial h_s}{\partial t} - n\,\delta \frac{\partial h_f}{\partial t} \right]$$
$$= \frac{\partial}{\partial x}\left(B_s K_{s_x} \frac{\partial h_s}{\partial x} \right) + \frac{\partial}{\partial y}\left(B_s K_{s_y} \frac{\partial h_s}{\partial y} \right) + Q_s \quad (2)$$

K_{f_x} and K_{s_x} (respectively K_{f_y} and K_{s_y}) are the hydraulic permeabilities in the fresh and salt water in x-direction (respectively in y-direction), h_f and h_s are the heads, B_f and B_s are the thickness of fresh and salt water zone, Q_f and Q_s are the fresh and salt water source/terms and n is the porosity.
We note also by $\delta = \dfrac{\rho_f}{\rho_s - \rho_f}$, where ρ_f and ρ_s are the specific weights in fresh and salt water, and by

$$\alpha = \begin{cases} 0 & \text{for confined aquifer} \\ 1 & \text{for unconfined aquifer} \end{cases}$$

Invoking continuity of the pressure at the interfaces, the interface elevation can be calculated from the fresh water and salt water heads by

$$\zeta = (1+\delta)\, h_s - \delta\, h_f \quad (3)$$

The equations (1) and (2) represent two coupled, parabolic partial differential equations that should be solved simultaneously for the fresh water head h_f and salt water head h_s. Once these values are known, the interface elevation ζ can be obtained from (3).
The set of the equations (1) and (2) can be written in the vectorial conservative form (Bouzouf et al., 1999):

$$\frac{\partial W}{\partial t} - \frac{\partial R_x(W)}{\partial x} - \frac{\partial R_y(W)}{\partial y} = S(W) \quad (4)$$

with

$$^tW = (S_f B_f + n(\alpha+\delta)h_f - n(1+\delta)h_s, \; S_s B_s + n(1+\delta)h_s - n\delta h_f)$$

$$^tR_x(W) = (B_f K_{f_x} \frac{\partial h_f}{\partial x}, \; B_s K_{s_x} \frac{\partial h_s}{\partial x})$$

$$^tR_y(W) = (B_f K_{f_y} \frac{\partial h_f}{\partial y}, \; B_s K_{s_y} \frac{\partial h_s}{\partial y})$$

$$^tS(W) = (Q_f, \, Q_s)$$

FINITE VOLUME DISCRETIZATION

To solve the system of equations (4) we have considered a triangular cell-centered finite volume formulation (Elmahi et al., 1999), where the state variables W_i^n are the average values for the cells at time level n:

$$W_i^n = \frac{1}{Area(C_i)} \int_{C_i} W(x,y,t^n)\, dx\, dy$$

Integration of the system (4) over a control volume C_i yields in explicit formulation:

$$W_i^{n+1} = W_i^n + \frac{\Delta t}{Area(C_i)}$$
$$\left[\int_{\partial C_i} [R_x(W^n) n_x + R_y(W^n) n_y]\, d\sigma \right] + \Delta t S(W_i^n)$$

where Δt is the time step and n_x and n_y are the components of the outward unit normal to ∂C_i. The discretization of equation (5) requires the approximation at each inteface Γ_{ij} separating two cells C_i and C_j of terms such as

$$\int_{\Gamma_{ij}} \left(B_l K_{l_x} \frac{\partial h_l}{\partial x} \right) n_x d\sigma \;, \; \int_{\Gamma_{ij}} \left(B_l K_{l_y} \frac{\partial h_l}{\partial y} \right) n_y d\sigma$$

where the index l refers here to the fresh or salt water. Coudiere et al (Coudiere et al., 1996), have studied an elliptic problem

$$\begin{cases} -div(A \nabla u) = f & \text{in } \Omega \subset I\!R^2 \\ u|_\Gamma = g & \text{over } \Gamma = \partial\Omega \end{cases} \quad (5)$$

Where A is a symmetric definite positive matrix with coefficients a_{ij} in $C^1(\Omega)$, $f \in C^0(\Omega)$ and $g \in C^2(\Gamma)$. They have used a Green-Gauss type interpolation to construct the gradients at the interfaces of the mesh. The gradient on each edge is approached by the Green theorem and then a first order Gauss quadrature formula, for which requisite values at the vertices are interpolated from the states on their neighbourhood .
We took inspiration from this scheme for devising our numerical procedure and discretize the diffusive contributions. We begin firstly by writing

$$\int_{\Gamma_{ij}} \left(B_l K_{l_x} \frac{\partial h_l}{\partial x} \right) n_x\, d\sigma$$
$$= (B_l K_{l_x})_{|\Gamma_{ij}} \frac{\partial h_l}{\partial x}|_{\Gamma_{ij}} \int_{\Gamma_{ij}} n_x\, d\sigma$$

One constructs the co-volume C_{dec} centered at the interface Γ_{ij} and connecting the barycenters G_i and G_j of the triangles that share this edge and the two endpoints N and S of Γ_{ij} (see figure 1).

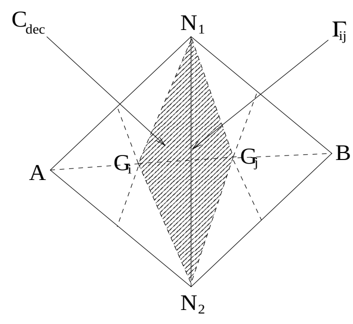

Figure 1: Diamond shaped co-volume

To calculate $\frac{\partial h_l}{\partial x}|_{\Gamma_{ij}}$, the divergence theorem is applied to the co-volume C_{dec} surrounding Γ_{ij}, which gives

$$\frac{\partial h_l}{\partial x}|_{\Gamma_{ij}} \simeq \frac{1}{Area(C_{dec})} \sum_{\varepsilon \in \partial C_{dec}} h_l|_\varepsilon \int_\varepsilon n_{x\varepsilon}\, d\sigma$$

ε represents an edge of the co-volume C_{dec} and $n_{x\varepsilon}$ is the axial component of the outward unit normal to ϵ. If we note by $\varepsilon = [N_1, N_2]$, One can write also

$$\frac{\partial h_l}{\partial x}|_{\Gamma_{ij}} \simeq \frac{1}{Area(C_{dec})} \sum_{\varepsilon \in \partial C_{dec}} \frac{1}{2}(h_{l\,N_1} + h_{l\,N_2}) \int_\varepsilon n_{x\varepsilon}\, d\sigma \qquad (6)$$

where $h_{l\,N_1}$ and $h_{l\,N_2}$ are respectively the values of h_l at the nodes N_1 and N_2 of the edge ε.
The data at the centers G_i and G_j are known exactly while the data at the vertices N and S must be determined by a interpolation procedure. For one node P of the mesh, one utilizes a linear approximation v of h_l on the set of cells which share the vertex P. One writes

$$h_{l\,P} = \sum_{K \in V(P)} \alpha_K(P) h_{l\,K}$$

where $V(P)$ is the set of triangles K surrounding P, $h_{l\,K}$ the head at the center of triangle K and $\alpha_K(P)$ are the weights of the interpolation corresponding to the node P.

MODEL VALIDATION

To verify and validate the numerical solution obtained from the finite volume model, numerical simulations have been compared to existing analytical solutions (Bear et al., 1999).

Steady state

Two cases have been checked: confined and unconfined aquifer, for both of them the initial values of h_f and ζ are arbitrarily fixed (Jose et al., 1983). The analytical solutions are as follows:

- Unconfined aquifer ((Veruijt, 1968), (Vappicha et al., 1975)):

$$\zeta(x) = -\left(\frac{2q_0\delta^2 x}{(1+\delta)K} + \frac{(\delta-1)}{(\delta+1)}(\frac{\delta\beta q_0}{K})^2\right)^{1/2}$$

$q_0 = 10$ m^2/day , $\beta = 0.741$, K=60 m/day

- Confined aquifer ((Glover, 1959), (Rummer et al., 1962)):

$$\zeta(x) = Z^t - \left(\frac{\mathbf{2}q_0 x}{\delta K} + (\frac{\delta\beta q_0}{K})^2\right)^{1/2}$$

$q_0 = 5$ m^2/day , $Z^t = -10$ m.

Unsteady state

Keulegan (Keulegan, 1954) presented an analytical solution for the interface in a confined aquifer of uniform thickness:

$$\zeta(x,t) = \frac{D}{2}\left\{1 + \frac{x}{[(\Delta\rho K\,Dt)\,/\,(n\rho_f)]^{1/2}}\right\}$$

D=10 m , n= 0.3 , K=39.024 m/day. The numerical solutions are in good agreement with the analytical solutions as depicted in figures (2), (3), (4), (5) and (6).

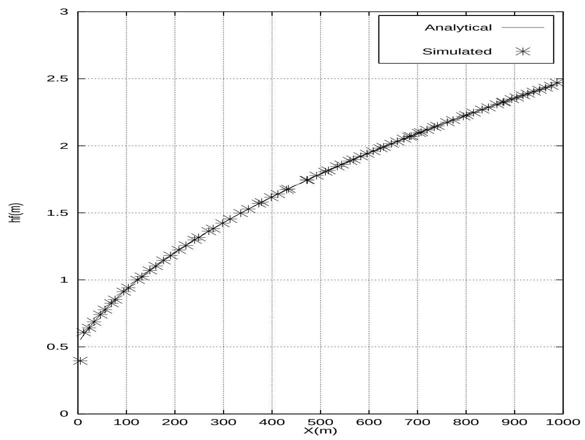

Figure 2: Comparison between analytical and numerical solutions of the head for unconfined aquifer (steady state).

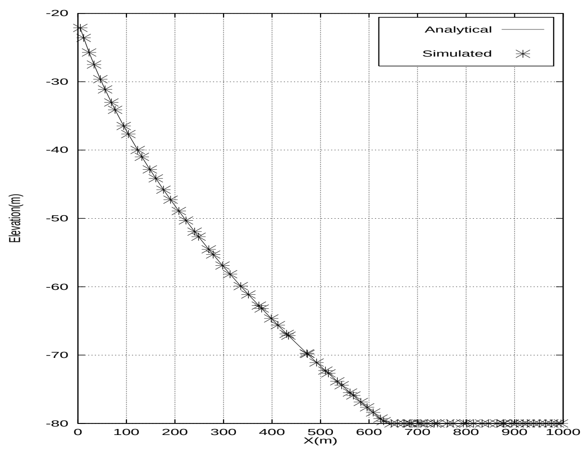

Figure 3: Comparison between analytical and numerical solutions of the interface elevation for unconfined aquifer (steady state).

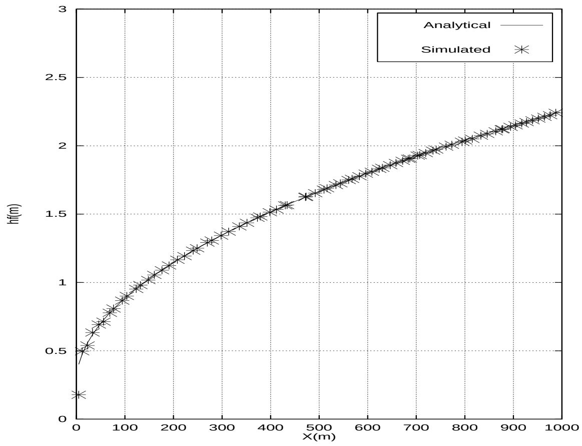

Figure 4: Comparison between analytical and numerical solutions of the head for confined aquifer (steady state).

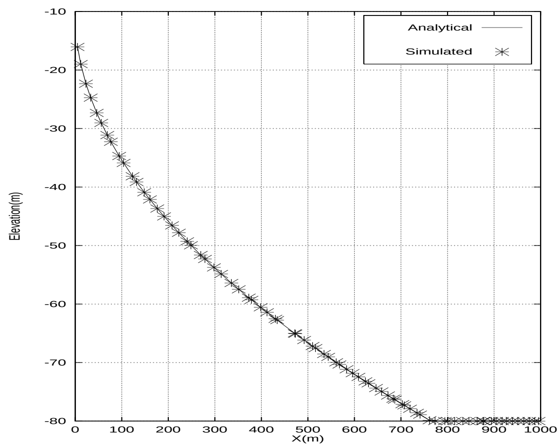

Figure 5: Comparison between analytical and numerical solutions of the interface elevationfor confined aquifer (steady state).

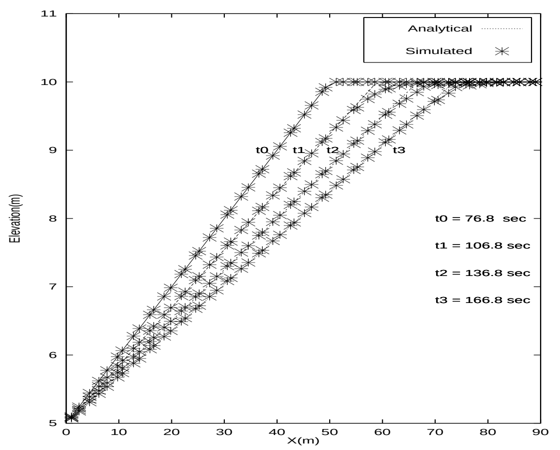

Figure 6: Comparison of analytical and simulated solution for unsteady state case.

APPLICATION TO THE LLOBREGAT DELTA, SPAIN

The Llobregat delta, extends over about $100\ Km^2$ (see figure(7)). It belongs to the structural domain of the Spain Mediterranean plain.

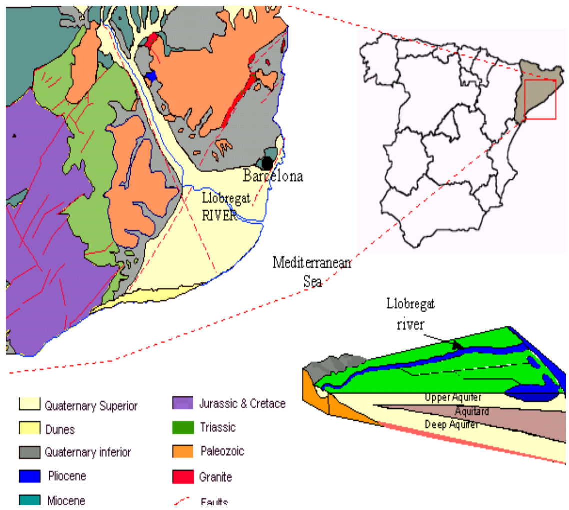

Figure 7: Geological map of the llobregat delta.

The finite volume based model has been applied to Llobregat delta. The initial conditions are taken on 1994 and result from the geophysical results. It is assumed that the boundaries of the aquifer are impermeable except at the coast, constants heads are imposed here. The simulation period is from 1994 to 1998. The model values of aquifer parameters, conductivity, storativity, porosity and thickness of the aquifer, was initially chosen on the basis of available data and was calibrated using hydrogeochemical and the geophysical data (Himi et al. 1998). In figure (8), the mesh of this aquifer is presented. Figure (9) illustrates the comparison between numerical and geophysical results. Close agreements are achieved.

CONCLUSION

Characterization of some coastal aquifer systems may be accomplished by assuming that salt water and freshw ater are separated by a sharp interface. Invoking the Dupuit assumption and performing a vertical integration results in quasi-three-dimensional, the equations may be solved to give fresh water head, salt water head and interface elevation. Cell-centered finite volume scheme on an unstructured mesh is used to approximate the partial differential equations. Comparisons of the finite volume approach adopted in this paper, with known analytical solutions and geophysical results have shown close agreement .

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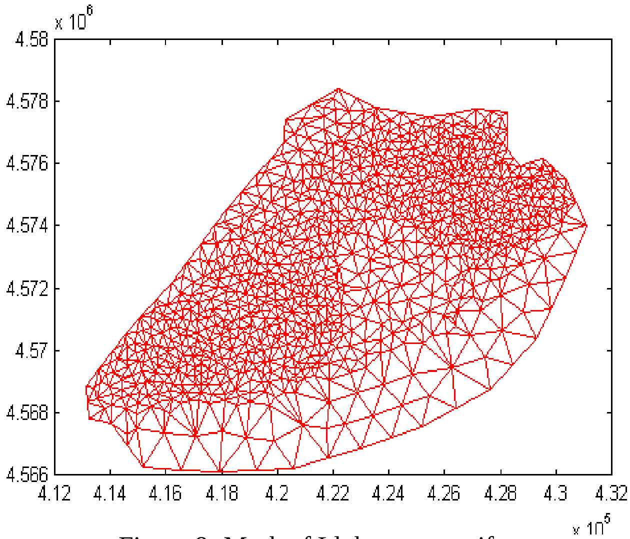

Figure 8: Mesh of Llobreagat aquifer

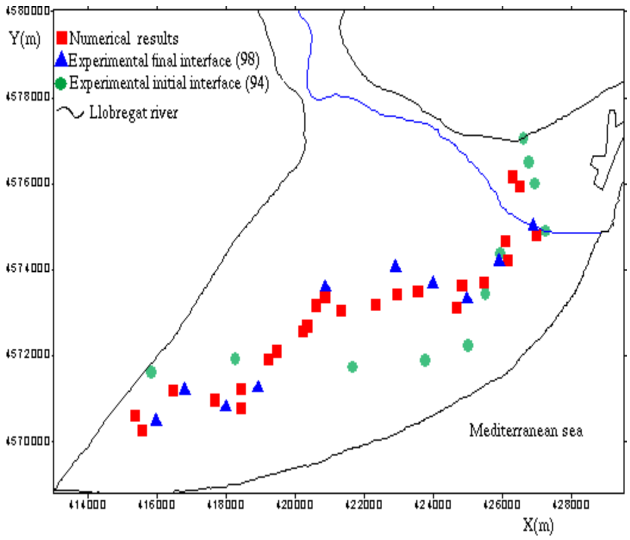

Figure 9: Comparison between experimental results and finite volume method in Llobregat aquifer.

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