VERIFICATION AND VALIDATION OF AN OPTIMIZATION MODEL FOR GROUNDWATER DEVELOPMENT IN COASTAL AREAS

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Abstract

Generally, an optimal model consists of simulation and optimisation models. A few optimal models have been developed and applied to control contamination in effective groundwater development. Also, there are several pumping models that consider the characteristics of coastal aquifers, such as density-variable flow and saltwater intrusion.

A new optimal pumping model has been developed. It evaluates optimal groundwater withdrawal and the location of a pumping well in the steady-state condition while minimizing adverse impacts, such as water quality, draw-down, saltwater intrusion and upconing. The objective of this study is to verify and to validate the optimal pumping model. The model is verified using a hypothetical unconfined aquifer, and is validated using data collected from sand tanks. The optimal solutions are verified by examining the behaviour of objective function values and adverse impacts in accordance with locations. Secondly, freshwater/saltwater interface and salinity obtained from both experiments and numerical studies are compared to validate the optimal pumping model.

Keywords: Optimal pumping model, coastal aquifer, sand-container experiments, saltwater intrusion

Introduction

The average annual precipitation in Korea is 1,283 mm, with a total volume of \(1.276 \times 10^3\) billion m\(^3\). But the total amount of annual precipitation per capita is just \(2,705\) m\(^3\), which is about 10% of the world average (MOCT, 2001). This is due to high population density in Korea. Furthermore, the dependence on groundwater in coastal areas is about four times larger than that of the national average due to insufficient
surface water and/or inadequate water distribution infrastructure (Hong et al., 2003). Therefore, imprudent groundwater use in coastal areas continues without consideration for environmental components such as saltwater intrusion and the pollution of wells. The present model considers these problems in its assessment of groundwater development and management.

Optimization models have been applied to a wide range of water resource management problems. (Belaineh, 1999; Guan and Aral, 1999; Sun and Zheng, 1999; Morshed and Kaluarachchi, 2000; Chang and Hisiao, 2002; Hsu and Chang, 2002; Greenwald, 2003; Ko and Lee, 2002; Kuo and Liu, 2003; Prasad et al., 2003; Samuel and Jha, 2003; Zheng, 2003; among others). An optimization model generally consists of simulation models and optimization techniques to obtain an optimal solution when facing specific problems. A simulation model is selected according to the characteristics of a given problem, and the optimization technique is selected according to the mathematical characteristics of the optimization problem.

To date, many optimization models have been developed and applied to coastal aquifers considering saltwater intrusion (Table 1).

### Table 1. Published optimal models

<table>
<thead>
<tr>
<th>Optimization Method</th>
<th>Decision Variable</th>
<th>Author, Country</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structured messy GA</td>
<td>Pumping rate</td>
<td>Cheng et al., USA</td>
<td>2000</td>
</tr>
<tr>
<td>Outer approx. method</td>
<td>Pumping rate</td>
<td>Papadopoulou et al., USA</td>
<td>2000</td>
</tr>
<tr>
<td>Bundle trust method</td>
<td>Net benefit</td>
<td>Gordon et al., USA</td>
<td>2000</td>
</tr>
<tr>
<td>GA</td>
<td>Construction cost</td>
<td>Mundzir, Canada</td>
<td>2001</td>
</tr>
<tr>
<td>LP</td>
<td>Cost</td>
<td>Hailu, USA</td>
<td>2002</td>
</tr>
<tr>
<td>Simple GA</td>
<td>Benefit &amp; cost</td>
<td>Benhachmi et al., Morocco</td>
<td>2003a</td>
</tr>
<tr>
<td>Simple GA</td>
<td>Pumping rate</td>
<td>Benhachmi et al., Morocco</td>
<td>2003b</td>
</tr>
<tr>
<td>Response matrix method</td>
<td>Pumping rate</td>
<td>Motz et al., USA</td>
<td>2003</td>
</tr>
<tr>
<td>GA, SA, TS</td>
<td>-</td>
<td>Peralta, USA</td>
<td>2003</td>
</tr>
<tr>
<td>Evolutionary optimization</td>
<td>Cost</td>
<td>Silva et al., Portugal</td>
<td>2003</td>
</tr>
<tr>
<td>Response matrix method</td>
<td>Pumping rate</td>
<td>Zhou et al., China</td>
<td>2004</td>
</tr>
<tr>
<td>PGA</td>
<td>Pumping rate &amp; well location</td>
<td>Park and Aral, USA</td>
<td>2004</td>
</tr>
<tr>
<td>Parallel GA</td>
<td>Pumping rate (FW &amp; SW) &amp; well location</td>
<td>Park et al., Korea</td>
<td>2003 a,b</td>
</tr>
</tbody>
</table>

However, most of the optimization models concern only one decision variable, either optimum pumping rates or minimum costs. In most real problems, there are two important questions besides cost: where a pumping or injection well should be installed, and how much groundwater can be withdrawn from the well.

Optimization models that can consider optimal pumping rates and locations were developed by Guan and Aral (1999), and Park and Aral (2004). A new model presented in this paper differs from those of Guan and Aral (1999), and Park and Aral (2004) in that it can handle more diverse problems. The new model can solve saltwater pumping problems for protecting freshwater pumping wells. A two-phase numerical model is used to simulate freshwater and saltwater flows. A parallel genetic algorithm is used to take advantage of a PC cluster and to expedite the solution process of the optimization model. The new optimization model is verified and validated against experimental results obtained from laboratory sand tanks.
Groundwater optimal management model

Numerical model for regional groundwater in coastal areas

Groundwater flow in coastal aquifers is somewhat different from that of inland aquifers because of the proximity of the sea. The optimal pumping model includes a simulation model that can consider freshwater and saltwater flows in coastal areas. The mathematical model used in this model is based on two vertically integrated governing equations, one describing freshwater flow and the other describing saltwater flow in an aquifer layer.

\[
\nabla \cdot (b_i \bar{K}^F \cdot \nabla h^f) = b_i S_i \frac{\partial h^f}{\partial t} - \frac{\partial \bar{z}^f}{\partial t} - Q^f
\]

\[
\nabla \cdot (b_i \bar{K}^S \cdot \nabla h^s) = b_i S_i \frac{\partial h^s}{\partial t} + \frac{\partial \bar{z}^s}{\partial t} - Q^s
\]

In this two phase model, the rates of freshwater and saltwater extracted depend on the position of the interface with respect to the elevation of the screened interval of the well. The production rates of freshwater and saltwater are given by:

\[
Q^f = \left( \frac{K^f L^f}{K^f L^f + K^s L^s} \right) \times Q^T
\]

\[
Q^s = \left( \frac{K^s L^s}{K^f L^f + K^s L^s} \right) \times Q^T = Q^T - Q^f
\]

where, L is the length of screen in the well, Lf refers to the distance from the interface position to the top of the screen, Ls refers to the distance from the interface position to the bottom of the screen, and Q^T is the total production rate of the well.

Optimal groundwater management model

Modelling is generally performed under given conditions to assess the influences due to pumping for the different pumping locations and pumping rates. However, optimal well locations and pumping rates can be determined by an optimal pumping model without using the above procedure. The types of problems which will be discussed in this study have been detailed in Table 2.

This paper suggests a systematic model to obtain an optimal solution for a pumping rate or location. Groundwater development in coastal aquifers can be commonly classified into one of five types listed in Table 2. These can be treated with the following single objective function;

\[
\text{maximize } \Phi = a \sum_{i=1}^{N_{opt}} Q_f - \omega_1 \sum_{i=1}^{N_f} \frac{Q_s}{Q_f + Q_s} - \omega_2 \sum_{i=1}^{N_s} Q_s
\]

where Q_f and Q_s are the freshwater and saltwater pumping rates in the well i, N_{opt} is the number of targeted optimal wells, N_p is the total number of wells including both optimal wells and existing wells in a domain, N_s is the number of wells designed to withdraw saltwater, \( \omega_1 \) and \( \omega_2 \) are the weighting factors,
and \( \alpha \) is a groundwater protection index representing the adverse impacts in the groundwater environment caused by pumping. The number of wells is assumed to be determined a priori.

The first term can be considered a benefit, the second term is a penalty function which decreases the value of the objective function when saltwater is pumped from the wells designed to pump freshwater only, and the third term is the cost incurred in saltwater pumping to control the saltwater wedge. The schematics of adverse effects are shown in Figure 1.

### Table 2. Types of problems studied

<table>
<thead>
<tr>
<th>Stage 1: Optimal Arrangement of the wells</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A priori Determined Variables</strong></td>
</tr>
<tr>
<td>CAT1A</td>
</tr>
<tr>
<td>CAT1B</td>
</tr>
<tr>
<td>CAT2A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stage 2: Control of the Saltwater Wedge</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A priori Determined Variables</strong></td>
</tr>
<tr>
<td>CAT2B</td>
</tr>
<tr>
<td>CAT4</td>
</tr>
</tbody>
</table>

Figure 1. Adverse impacts of pumping in coastal areas
The groundwater protection index is defined as below;

\[ \alpha = 1 - (D + I + A) \times \omega_3 \]

where, \( \omega_3 \) is a weighting factor, \( D \) is weighted normalized average drawdown \((D = V_D / A_T H)\), \( I \) is weighted normalized average increase in saltwater thickness \((I = V_S / A_T H)\), \( A \) is weighted normalized saltwater volume \((A = A_I / A_T)\), \( A_T \) is the domain area, \( V_D \) is the volume of drawdown, \( H \) is the saturated thickness, \( V_S \) is the change in saltwater volume, and \( A_I \) is the change in intrusion area.

Parallel genetic algorithm

A genetic algorithm has the capability to search for a global optimum (Goldberg, 1997). However, the method has large computational requirements. In this work, the large computational requirements can be efficiently handled with parallel processing. The GA is well suited for parallel processing. Figure 2 depicts a schematic diagram of the parallel computation. A PC cluster, composed of 32 processors, is used in this study.

Verification of the optimal pumping model in cross section

Situation 1: Optimal \( Q_f \) for a well with fixed location

To verify the optimal pumping model, a hypothetical cross-sectional unconfined aquifer is used (Figure 3).
A partial penetrating well is installed at x = 250 m. In this case, the question would be how much we can pump from this well? Since the problem is simple cross-sectional, the answer can be found via simple trial and error. Therefore, the optimal solution can be verified in a straightforward manner. Execution of the optimization model resulted in the pumping rate of 0.762 m$^3$/day. The freshwater-saltwater interface according to the pumping rate is depicted in Figure 4.

![Figure 4. Optimal pumping rate in an arbitrary well](image)

As it can be seen, the interface is in close proximity of the bottom of the well screen. Distance between the interface and the well can be constrained to ensure more safety.

**Situation 2: Optimal Q$_f$ and location**

Another type of question may be where to put the well and how much groundwater to pump. The proposed optimization can handle this problem. If there were no constraint on pumping, the well would be located at the inland end of the domain. But, if impact on the groundwater environment is a concern, it becomes a trade-off question and must be formulated as a multi-objective optimization problem. In this study a simple approach is taken: weights are used to convert multi-objectives into a single objective. For the weight used in the study the optimal well location and pumping rate are 400 m and 0.864 m$^3$/day, respectively (Figure 5).

![Figure 5. Optimal pumping rate and well location](image)
The optimality of the solution can be verified by determining optimal pumping rate for a well placed at every location (Figure 6). It is clear from the figure that pumping rates can be increased as the well is moved towards the inland boundary.

However, the objective function value stops increasing after the optimal location (x = 400 m) since the ground water protection index decrease (Figure 7). It must be noted that the objective function value depends on the weight. The effects of \( \omega_3 \) are depicted in Figure 8.

As the value increases, the optimal point moves toward the coastal boundary. That is to say, the optimal solution may vary depending on the importance of groundwater environment or groundwater development.

**Situation 3: Optimal Q_s and location (control of saltwater wedge)**

There are two common methods to control and/or retard saltwater wedge. The first method is the injection of freshwater to create a groundwater mound. The second method is the extraction of saline groundwater. In this section, an optimal solution of the second method is studied to control saltwater wedge.

Let’s assume that 10% more groundwater than the optimal pumping rate suggested in the previous section is required. As the required freshwater pumping rate is higher than the optimal pumping rate, a control
measure is necessary to protect the well from saltwater intrusion. It turns out that if saltwater is pumped at 0.106 m³/day, the freshwater well can be protected. The interface position is depicted in Figure 9.

An optimal solution is in accordance with the minimal saltwater pumping rate (Figure 10). As the location of saltwater pumping come near to the coast line, the saltwater pumping rate should be increased to secure the required fresh groundwater. So, it is a natural result because the increase of saltwater pumping rate causes the decrease of the objective function value. Accordingly, the capability of the optimal pumping model that controls the saltwater wedge is also verified.

**Validation of the optimal pumping model using sand tank experiments**

**Cross-sectional flow (freshwater lens)**

**Situation 1: Optimal Q₁**

The conceptual model is depicted in Figure 11. The size of the small sand tank is 0.43 m long, 0.08 m wide and 0.30 m high. The intrinsic permeability of the sand is 1.092 x 10⁻¹⁰ m². Two saltwater supply chambers are on both sides of the sand tank. The density of saltwater is 1040 kg/m³ and the depths of water in both chambers are equal at 0.182 m. Six mL/min of freshwater are injected at the center (x = 0.22 m) by the variable speed peristaltic pump.

The experimental work is performed using the results of the optimization model, and the optimization model is validated by comparing observed and calculated results. The optimal pumping rate for the well installed at x = 0.12 m, calculated by the optimal pumping model, is 2.4 mL/min. The comparison of the
computed and observed interfaces is presented in Figure 12. Except for the center position the agreement is very good.

The calculated interfaces and the observed one are relatively inexact in the center of the sand tank. This is why the recharge condition is given as an injection well in numerical analysis. But the results of interface position are entirely in accord with each other and there is no pollution in the pumping well.

**Situation 2: Optimal Qₜ (control of saltwater wedge)**

The injection rate is same as in situation 1. The pumping rate is raised to 3.5 mL/min at 0.12 m. The well is contaminated: calculated and observed percentages of saltwater are 4.25% and 3.4%, respectively. Even though the two-phase model is used, water quality can be modeled with reasonable accuracy.

As a second validation problem, the amount of saltwater pumping to protect the freshwater pumping well is computed and tested with experiment.

The optimal saltwater pumping rate is computed as 3 mL/min. After pumping the saltwater, 0.05% (salinity = 118 ppm) of the pumping rate is observed as saltwater. Figure 13 shows the results.
Verification and validation of an optimization model for groundwater development in coastal areas

**Figure 12.** Results and picture of situation 1 (Optimal $Q_f$)

**Figure 13.** Results and picture of situation 2 (Optimal $Q_s$)
Dimensional flow (lateral intrusion)
Situation 1: Optimal $Q_f$

The size of the 3-dimensional sand-container is 2.0 m long, 1.6 wide and 0.6 m high. The thickness of sand layer is approximately 0.5 m. There are two water supply chambers on both sides of container, one is filled with freshwater and the other is filled with saltwater. the water level of the freshwater chamber is 1.8 cm higher than that of the saltwater chamber. There are two pumping wells. One well is assumed an existing well at $x = 0.9$ m, $y = 0.8$m. Its pumping rate is 15 mL/min. The second well at $x = 1.2$ m, $y = 0.5$m; this is the well where the pumping rate is to be optimized (Figure 14). A 4x7 array of observation wells (Figure 14) are installed to measure the interface position. Measurements of the interface needs special care and will be presented separately elsewhere.

An optimal pumping rate in the new well is calculated at 27 mL/min. This value is the maximum pumping rate in the new well while protecting both wells from saltwater intrusion. Experiments are performed according to the computed results. The calculated interface agrees closely with the freshwater side limit of the observed transition zone.
Figure 15. Comparison of the observed interface and the calculated interfaces

Conclusions

An optimization model is numerically verified and experimentally validated. The model is unique in the sense that it can solve not only for optimal pumping rates and well locations, but also freshwater injection and saltwater pumping rates to protect freshwater pumping wells. This work is also unique in the sense that it is the first experimental validation of an optimization model, to the best knowledge of the authors. Major findings from this study are:

- A quantitative decision support tool is proposed for best management practices for groundwater development in coastal areas.
- The two phase model can predict the quality of pumped water with reasonable accuracy.
- The interface from the two-phase model agrees with the freshwater side limit of the transition zone.
- The capability of the simulation and optimization model is verified and validated.

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References


Topic 2.

Modelling of saline water intrusion