INVERSE MODELING FOR COASTAL SEAWATER INTRUSION: PARAMETER SENSITIVITIES, VARIANCES, CORRELATIONS AND ESTIMATION IN NOISELESS THEORETICAL PROBLEMS

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Abstract

Inverse modeling, also called parameter estimation, is becoming a standard tool that groundwater hydrologists apply when using a numerical groundwater model. Typically, inverse modeling has been applied to groundwater flow problems with fewer attempts to use the technique for modeling of heat or solute transport. The use of inverse modeling for variable-density fluid flow and solute transport, such as occurs in seawater intrusion into aquifers, is even more uncommon.

Inverse modeling of variable-density groundwater systems is accomplished by coupling the USGS SUTRA (Version 2D3D.1) groundwater code (Voss and Provost, 2002) and the UCODE (Version 3.05) inverse code (Poeter and Hill, 1998). Insight into practical difficulties and general guidance concerning techniques are gained from the analyses of a basic 2D-inverse problem: the classic Henry variable density groundwater model benchmark.

Keywords: inverse modeling, seawater intrusion, sensitivities, network design

Objectives and approach

The objective of this analysis is twofold:

1. To evaluate the ability of inverse modeling to determine parameters of seawater intrusion models and to develop techniques applicable to inverse modeling of such systems.
   a. To test the ability of inverse modeling to determine values of various combinations of model parameters that a practitioner may attempt to estimate for a field problem (when the model has too many estimated parameters or has several non-independent estimated parameters).

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b. To attempt to employ basic estimation statistics as a means to reduce the number of parameters to estimate and to identify sets of independent parameters.

2. To study the value of various types and locations of field data in seawater intrusion situations in reducing the variance and correlation of parameter estimates.

Analyses are primarily concerned with the aspects of inverse modeling that are dependent on the physics of the problem, not on errors in data or on errors in model features. Therefore, data generated by forward simulation of the problems is employed as input for inverse runs without adding noise.

Discussion and conclusions

Reduction of the number of estimated parameters is a useful strategy for successfully solving inverse problems that may have poor estimation convergence, be non-unique, have multiple minima, or require excessive computational effort, conditions that often occur when estimating too many or codependent parameters. A stepwise reduction approach that relies on physical logic and correlation-covariance analysis is recommended. All parameters have some natural correlation resulting from the physics of the problem, because they affect the observed values of pressure or concentration in similar (or opposite) ways in some parts of the domain considered. However, parameters with correlations near ±1 cannot be estimated simultaneously and must be removed from the estimation process by fixing their values or by linking their values to other estimated parameters. In addition, stopping the estimation process and resetting initial guesses for some parameters is sometimes necessary to obtain proper convergence of the estimation and to lessen the chance that the converged solution is not globally optimal.

The Henry problem (Henry, 1964 and Ségol, 1994) is the oldest semi-analytical solution of a steady-state seawater intrusion problem containing a dispersed transition zone between freshwater and seawater ϑ. The Henry solution represents the mechanical dispersion of solute in a rectangular confined aquifer in cross section as a pure diffusion process with diffusivity. The system presents a freshwater inflow from the regional aquifer system on one end and contact with the ocean on the other. Henry solution shows that the parameters governing equations for seawater intrusion affect the steady-state system physics only as arranged in three groups, a, b and ξ, defined as follows:

$$a = \frac{Q\mu}{k(\rho_s - \rho_0)gd} \tag{1}$$

$$b = \frac{\varepsilon\vartheta}{Q} \tag{2}$$

and

$$\xi = \frac{l}{d} \tag{3}$$

where d is the aquifer thickness, g the acceleration of gravity, k the permeability of the aquifer, Q the net fresh-water discharge per unit length of coast, ε the porosity of the aquifer, ϑ the molecular diffusivity of solute in pure solid, ρ_0 and ρ_s the density of fresh- and seawater, μ the viscosity of water and l the aquifer length. The third parameter, called aspect ratio as it concerns the domain dimensions, is considered only as a fixed value in this study.

Taking the Henry problem as a rough representation of a typical seawater intrusion situation in a single aquifer, the ideal places to measure pressure are as far away from the coast as possible, and the ideal places to measure concentration are near the bottom of the aquifer between the center of the transition zone and its inland fringe (see Figure 1). Note that the areas of higher sensitivity in Figure 1 (i.e. the darker ones) represent good locations to measure pressure or concentration.

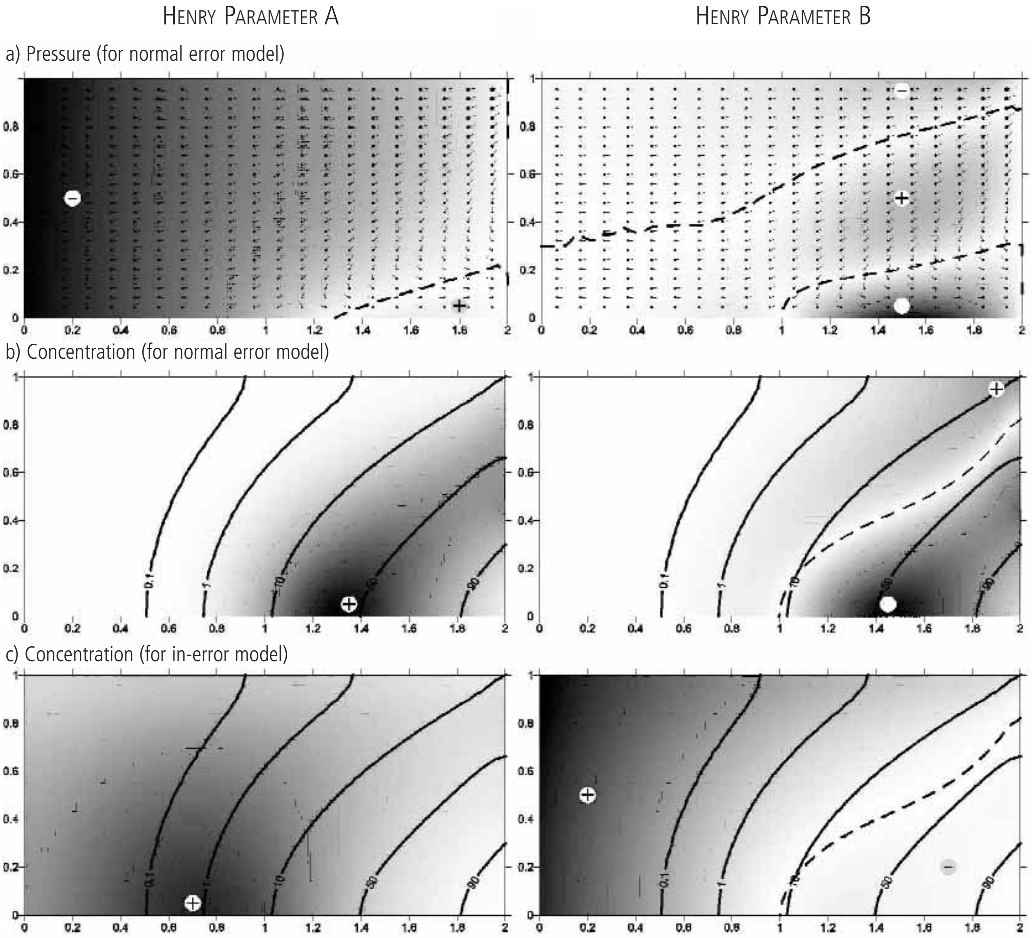

Figure 1. Maps of scaled sensitivity of a) pressure (for normal error model); b) concentration (for normal error model); and c) concentration (for ln-error model), to the Henry controlling parameter *a* and *b* (see the variables' definition in the text). The locus of zero sensitivity is marked with a dashed line. The groundwater flow field is also shown (flow is directed from the circular base marker located at each element centroid towards the end of the line). Black curves are isopleths of concentration equivalent to 0.1, 1, 10, 50 and 90% seawater. For each plot, white represents zero sensitivity and gray tones represent the absolute value of sensitivity where black represents the greatest absolute value. The sign of the sensitivity in a part of the map is also indicated. Sensitivities were calculated for a 1% change in each parameter.

These observations, in and near high-sensitivity regions, reduce parameter estimation variance. The error model that applies to the observations strongly affects the sensitivity distributions and so must be considered in effective sampling network designs. For the Henry problem, permeability and freshwater inflow can be estimated with low estimation variance from only pressure or only concentration observations. From observations of only the logarithm of concentration, permeability, freshwater inflow, solute diffusivity, and porosity can be estimated with roughly equivalent confidence.

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