Stochastic Analysis of Saltwater Intrusion in Heterogeneous Aquifers using Local Average Subdivision

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ABSTRACT

This study investigates the effects of ground heterogeneity, considering permeability as a random variable, on an intruding SW wedge using Monte Carlo simulations. Random permeability fields were generated, using the method of Local Average Subdivision (LAS), based on a lognormal probability density function. The LAS method allows the creation of spatially correlated random fields, generated using coefficients of variation (COV) and horizontal and vertical scales of fluctuation (SOF). The numerical modelling code SUTRA was employed to solve the coupled flow and transport problem. The well-defined 2D dispersive Henry problem was used as the test case for the method. The intruding SW wedge is defined by two key parameters, the toe penetration length (TL) and the width of mixing zone (WMZ). These parameters were compared to the results of a homogeneous case simulated using effective permeability values. The simulation results revealed: (1) an increase in COV resulted in a seaward movement of TL; (2) the WMZ extended with increasing COV; (3) a general increase in horizontal and vertical SOF produced a seaward movement of TL, with the WMZ increasing slightly; (4) as the anisotropic ratio increased the TL intruded further inland and the WMZ reduced in size. The results show that for large values of COV, effective permeability parameters are inadequate at reproducing the effects of heterogeneity on SW intrusion.

INTRODUCTION

It has been well established that heterogeneity, in the form of variable conductivity fields, has a significant effect on flow through a porous media (Held et al. 2005; Abarca, 2006; Ahmed, 2009; Kerrou & Renard, 2010). A popular method of representing heterogeneity is to use probabilistic methods. It has become so popular that Eurocode 7 recommends probabilistic methods as an option to consider when determining characteristic properties of porous media, as they take account of spatial variability (Hicks & Samy, 2002). This study seeks to comprehensively investigate the effects of heterogeneity by simulating a broader range of cases not reported in other works. The additional simulations provide results at the extremities of the known trends and solidify the findings of previous studies.

Background

Numerous investigations into heterogeneous saltwater intrusion have used a stochastic approach to account for the spatial variation in conductivity (Held et al. 2005; Abarca, 2006; Kerrou & Renard, 2010). Held et al. (2005) studied the effects of heterogeneity on an intruding saltwater wedge for the benchmark Henry problem. The objective of Held et al. (2005) was to determine whether results from heterogeneous stochastic simulations could be reproduced using simple effective parameters simulated homogeneously, thus neglecting the long computational time required for stochastic processes. Abarca (2006) recognised the limitations of the Henry problem and hypothesized a purely dispersive version that would better represent the salinity profiles seen in reality. The dispersive problem uses the same boundary conditions as the Henry problem but saltwater-freshwater mixing is assumed to be due to advection and velocity dependent dispersion, instead of diffusion. Kerrou & Renard

(2010) simulated this 2D dispersive Henry problem for isotropic and anisotropic cases and also extended the problem to the third dimension. This paper extends the 2D simulations of the dispersive Henry problem carried out by Abarca (2006) and Kerrou & Renard (2010) for cases of varying scales of fluctuation (SOF), coefficients of variation (COV) and anisotropic ratios (ξ), by combining the computer code SUTRA (Voss & Provost, 2010) with random field generation using local average subdivision (LAS) (Fenton, 1990).

METHODS

Random field generation using local average subdivision (LAS)

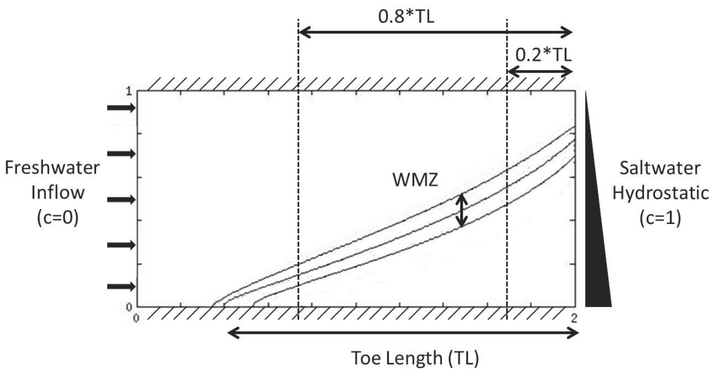
A lognormal distribution is used to describe the random permeability field $K(x)$ that has a mean (μ_k) and standard deviation (σ_k). Therefore $\ln(K)$ will have a Gaussian distribution where the respective mean ($\mu_{\ln k}$) and standard deviation ($\sigma_{\ln k}$) are determined from transformations (Fenton, 1990; Ahmed, 2009). The spatial correlation of the random fields takes the form of a Gauss-Markov exponentially decaying function, and is determined by the SOF (θ). The LAS algorithm creates a field of elements by dividing a global average value into 4 sub-fields whose local average is equal to the value of the global average (Fenton, 1990). This step continues recursively until the number of elements required in the field is achieved. Initially, LAS is used to construct a Gaussian random field, $G(x)$, with zero mean, unit variance and spatial correlation. The random permeability of each element is then assigned using the transformation:

$$K_i = \exp[\, \mu_{\ln k} + \sigma_{\ln k} G(x_i)\,] \tag{1}$$

where x_i is the centre of the ith element. Random fields in this study are defined by the scale of fluctuation (SOF,θ), coefficient of variation (COV $= \sigma_k/\mu_k$) and anisotropic ratios ($\xi = \theta_h/\theta_v$, where θ_h and θ_v are the horizontal and vertical SOF).

Dispersive Henry problem

The dispersive Henry problem consists of a confined aquifer with a hydrostatic saltwater pressure boundary at one side and a freshwater flux boundary at the other. Figure 1 shows the numerical domain and simulation inputs with visual representations of the TL and WMZ.



Input Parameters		
Geometric mean permeability	1.00E-09	m^2
Molecular diffusivity	0.00	m^2/s
Longitudinal dispersivity	0.10	m
Transverse dispersivity	0.01	m
Freshwater flux	6.60E-05	m/s
Freshwater density	1000	kg/m^3
Saltwater density	1025	kg/m^3
Dynamic viscosity	0.001	kg/m/s
Coefficient of variation	0.81, 1.31, 2.53, 4.37	
Horizontal scale of fluctuation	0.04, 0.12, 0.36	
Anisotropic ratios	1, 2, 3	
Realisations per simulation	120	

Figure 1. Dispersive Henry problem simulation domain and input parameters

The TL is defined as the distance between the saltwater boundary and where the 50% saltwater isoline intersects the bottom boundary. The WMZ is the average of the vertical distances between the 25% and 75% saltwater isolines within the range 0.2*TL and 0.8*TL. The TL and WMZ are determined for each realization and then averaged to give the values reported in this paper.

RESULTS

Effect of Coefficient of Variation (COV)

Results from a selection of the stochastic simulations are presented in Figure 2. When compared to the effective permeability simulations, an increase in COV resulted in a reduction of TL and an increase in WMZ for the heterogeneous simulations.

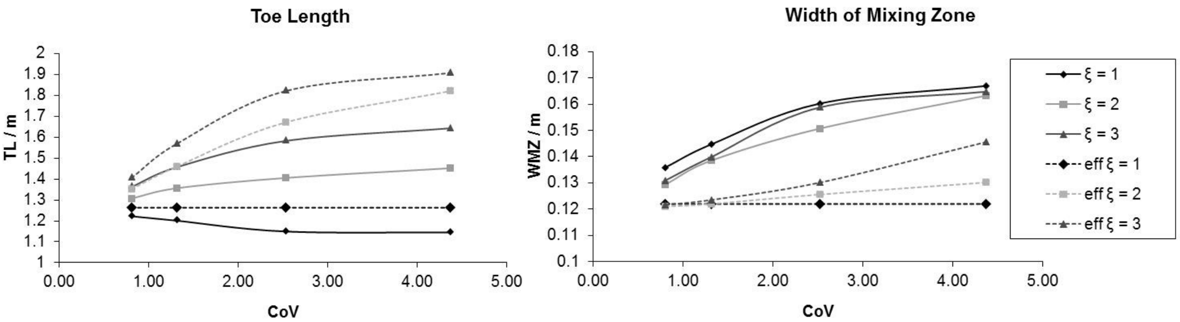


Figure 2. TL and WMZ for changing CoV and ξ for stochastic (line) and effective homogeneous (dashed) simulations

This was typical for all anisotropic ratios. The increase in WMZ can be explained by the greater longitudinal and transverse dispersions observed in heterogeneous media, which become greater with increasing COV. The effect of heterogeneity on TL can also be explained by the increase in dispersion. The increased dispersion acted to spread the salt further, reducing the sharpness of the density difference within the wedge. As a result the slope of the wedge became more linear than the curved shape observed in the homogeneous cases. The overall result was a reduction in TL with increasing COV. These results are in agreement with Abarca (2006) and Kerrou & Renard (2010).

Effect of Anisotropic Ratio (ξ)

As ξ increased the TL became larger, due to the flow pathways tending to align in the horizontal direction, forming channels of correlated permeability spanning the length of the domain. The chances of encountering a region of low permeability become smaller as the field becomes increasingly more correlated in the horizontal direction. This is observed most notably in the isotropic case (ξ = 1), where the TL decreased with increasing COV while it increased for anisotropic cases (ξ > 1). However both isotropic and anisotropic cases show a reduction in TL when compared to their effective homogeneous counterparts. The WMZ generally increased with increasing COV and only a slight decrease in width was observed with increasing ξ. This is because the WMZ is predominately determined by the dispersion, which will increase when there is more variability along the concentration gradient. However, dispersion also increases with higher fluid velocities which are observed in larger ξ in the horizontal direction. Therefore the change in WMZ with increasing ξ is small due to the balancing of dispersive mechanisms with regards to spatial variability and fluid velocity.

Effect of Scale of Fluctuation (θ)

In general, an increase in SOF resulted in a seaward movement of the TL, as is shown in Figure 3. For the smallest value of SOF shown (θ=0.04) the TL has intruded the furthest. This is contrary to what would normally be expected. An increase in SOF would imply an increase in dispersion, therefore resulting in a reduction of TL. However this trend can be explained in terms of the number of permeability flow channels available in the domain. As the SOF decreases, the porous media contains a larger frequency of high and low permeability zones. Whenever the flow encounters a low permeability zone, there is a higher chance that nearby a large permeability channel will exist and the flow will not have to deviate as much to continue moving further inland. Similar to the effect of changing ξ, the WMZ was not significantly affected by changes in SOF but was dominated by COV.

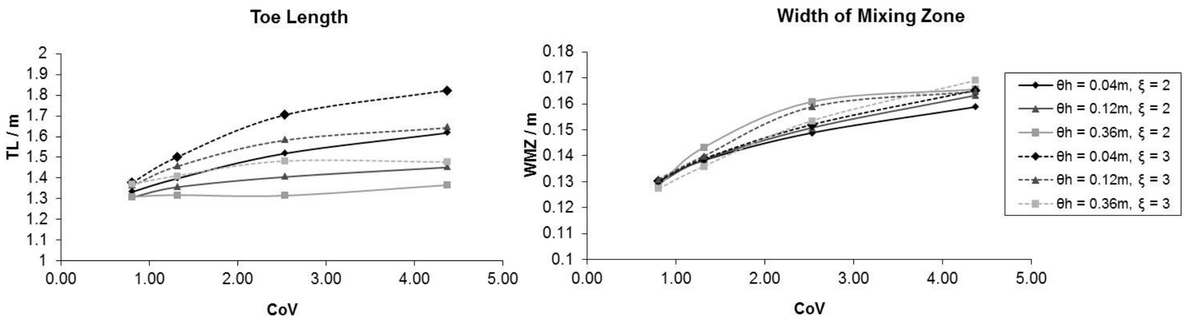


Figure 3. TL and WMZ for changing CoV, SoF and ξ

CONCLUSIONS

This paper has briefly described the effect of varying scales of heterogeneity on an intruding saltwater wedge. The scale of heterogeneity was represented by coefficients of variation (COV), scales of fluctuation (SOF) and anisotropic ratios (ξ). Simulations have shown that increases in COV dominated the changes in the width of the mixing zone, while the toe length movements were also dependent on ξ and SOF. An increase in ξ produced a landward movement of the toe, while an increase in SOF resulted in a seaward movement. Increases in COV resulted in enhancing the effects of these trends. The ability to apply an effective homogeneous permeability value to represent aquifer heterogeneity is useful as it negates the need for a computationally intense and time consuming stochastic process. However for large values of COV and SOF effective permeability values do not adequately portray the intrusion of a saltwater wedge in terms of toe penetration length and width of mixing zone.

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