Coupled hydrogeophysical inversion on synthetic example of seawater intrusion

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ABSTRACT

Seawater intrusion (SWI) is a complex process, where 3D modeling is often necessary in order to monitor and manage the effected aquifers. Unfortunately obtaining good quality groundwater data to support these models is difficult. Geophysics has become a common tool in the last two decades to supplement the lack of groundwater (GW) data. Geophysical methods are nonintrusive and less costly compared to standard drilling and offer an attractive alternative. Combining these two different sources of data, however, is still subject of ongoing investigation. One of the caveats is the different scales of geophysical and groundwater models, as well as the empirical petrophysical relationships that relate geophysical and groundwater states. Solving the parameter estimation problem in this field adds even more complexity, and careful analysis of the potential and the limitations of such an inverse problem should therefore precede the collection of field data.

We used Matlab to develop a 3D groundwater model for variable density flow, which is based on discretized flow and solute mass balance equations. In conjunction with the GW model, a geophysical model was developed for 3D electromagnetic (EM) modeling and inversion in the time domain. Having both models in the same environment gives space to implementing different coupling concepts.

Depending on the tightness of coupling between the two models in the inversion framework the approaches go from fully coupled framework to uncoupled approach, where the latter one was a focus of this work. With our models we can evaluate both sources of data at the same time, as well as within the inversion algorithms. In order to test the different coupling concepts for estimating the seawater intrusion process we started with a numerical test on a synthetic example. Seawater intrusion was simulated with our groundwater model code, for the geophysical application we use a time domain EM system with a loop source and receivers on surface. In our coupled framework the estimates of initial solute distribution served as a reference model for EM inversion and vice versa.

INTRODUCTION

The terminology differs among authors, usually as uncoupled inversion is meant inversion where geophysics and hydrogeology stay independent. Geophysical data are inverted to estimate the spatial distribution of some property, the outcomes are then converted with some petrophysical relationship, and then used as input data for the groundwater models. This is sometimes referred to as a sequential hydrogeophysical inversion. The big plus of this approach is that hydrogeological and geophysical models run independently, the disadvantage is that the geophysical inverse problem needs a regularization term, and therefore *a priori* information has to be entered in a form of smoother or a reference model.

By coupled approach we consider the framework where the geophysical and groundwater models are linked together during the inversion. For example, the GW model is often used as a form of regularization, providing realistic models for geophysical inversion. an example of field study with this approach is in Bauer-Gottwein (2009). In Herckenrath (2013) they further distinguish joint hydrogeophysical inversion when groundwater and geophysical model are simultaneously inverted.

In the following synthetic experiment we create a seawater intrusion scenario, with some propagation of saltwater front between time t_0 and t_1. In the inverse problem we then want to estimate previous solute fraction (at time t_0) using coupled framework with both sources of data, GW well data and time t_1, and EM data from t_0.

METHODS

Numerical models

A 3D groundwater model developed in Matlab is based on discretized flow and solute mass balance equations. Finite difference scheme was used for the pressure equation and Semi - Lagrangian method for solute transport equation. This enables us to choose an arbitrarily large time step without losing stability (up to some accuracy requirement) due to the coupled character of governing equations. We assumed steady state for groundwater flow; however the GW flow equation still has to be resolved throughout the computation to update the pressure and velocity field as a result of solute content dynamic. For the state equations the density dependency is considered in a linear form, and the viscosity is kept constant, not dependent on solute fraction or temperature. Both governing partial differential equations were discretized on a 3D staggered grid.

We derive analytical sensitivities of actual solute mass with respect to initial solute fraction and permeability based on the discretized governing equations. Analytically derived sensitivities not only reduce the computations cost of an inverse problem, but also give insight for maximizing information in collected data.

The geophysical model is based on the Maxwell's equations in the time domain; the quasi-static approximation can be used due to shutting of the source initially, and assuming low permittivity and small changes for electric field. The change of magnetic field was measured at given time steps for all receivers, which provides the geophysical data. The geophysical model is also discretized on a staggered grid, but has to be solved on a larger padded grid compared to groundwater model due to no flow boundaries of the EM model.

Archie's law was used for converting the solute fraction values to conductivity, assuming the knowledge of its parameters and that the bulk conductivity is affected only by electrolytic conductivity of water in the porous matrix and the surface conductivity of porous material is negligible. This or any other empirically based connection can be used for this inner coupling between the two models and enables anytime switch between the solute fraction and soil bulk conductivity in the idealized case. Generally the Archie’s law parameters are unknown but we can afford this luxury due to synthetic example setup.

Inversion

Both inverse EM and GW model were solved with a Gauss - Newton method. Since both inverse problems are ill-posed regularization has to be added. The objective function is:

$$\phi(m) = \phi_d + \beta\phi_m = \frac{1}{2}\left\|Q(d(m) - d_{obs})\right\|^2 + \frac{1}{2}\tau\left\|m - m_{ref}\right\|^2 + \frac{1}{2}\beta\sum_i \left\|G_{x_i}(m - m_{ref})\right\|^2$$

The *m* in EM model is soil bulk conductivity, in GW model is the solute fraction (saltwater content). The regularization parameters β and τ differs in each model as well as the data projection matrix Q. The weights of gradients (G_{xi}) in the regularization term are favoring the expected direction of groundwater flow in both models.

RESULTS

To set up the seawater intrusion synthetic experiment, we chose GW boundary conditions corresponding to simple Henry problem setup with heterogeneous permeability field and a pumping well. We run the model with a different parameter setup up to time t_0, which became an initial "unknown" solute content ω_0. The groundwater model then goes from time t_0 to a final state at time t_1, giving the "true" solute content ω_0 and ω_1. Two transects of few wells is placed along the flow direction in the east and west part of the domain and solute fractions are "measured" in some depth intervals at time t_1 (43 observation points). Next to that EM imaging was done at time t_1 with a large loop source in the center and receivers placed uniformly and densely on the surface over the area of interest (33 x 33 receivers).

Coupled inversion starts with EM inversion, the estimate of bulk conductivity at t_1 is transferred via Archie's law into ω_1, and serve as additional data for GW inversion with smaller weight and only in some data points across the area.
GW inversion then runs with this extra data and final estimate is again transformed into soil bulk conductivity via Archie's law to serve as a reference model for the next EM inversion The result of the EM inversion is then again an entry model for the GW inversion. This loop cycle can keep going as long as the estimate of initial/final solute fraction changes. In Fig.1 you can see the scheme. At the end we have an estimate of solute fraction ω_1 and bulk conductivity σ_1 at time t_1 as well as groundwater estimate of ω_0 at time t_0. Since this is a synthetic experiment we can also record the actual errors and not only the data misfit.

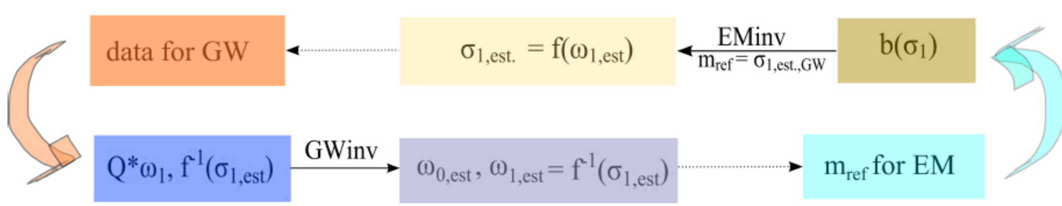

Figure 1. The coupled scheme used for inversion

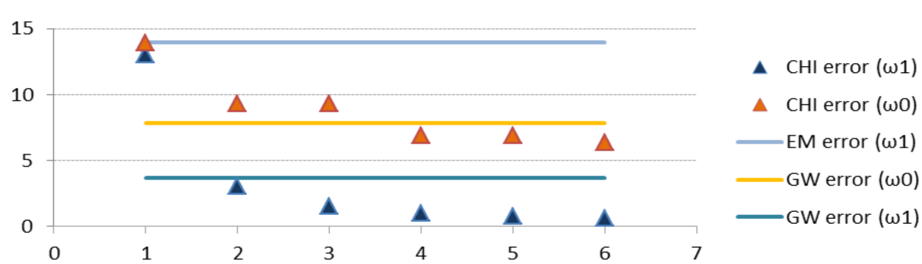

Figure 2. The actual error and data misfit decrease

After 3 runs of both GW and EM inversion we obtained estimate of the saltwater distribution at time t1 as it can be seen in Fig.3 (also the uncoupled GW inversion result). Fig.2 shows the decrease of the actual error in the estimate of ω_0 and ω_1 as well as the error of $\omega_{0/1}$ when only uncoupled inversion is applied.

CONCLUSIONS

The coupled inversion gave visually the best estimate of saltwater front shape and also produced the best actual error compared to poorly constrained EM or GW inversions. Adding more GW data makes the difference in the initial estimates smaller, in these cases we could say we have enough information in the GW model and the geophysical model can therefore just confirm its validity. We can expect similar results once the joint inversion is implemented, where weighting between the two data sets will probably have a strong effect on the final result.

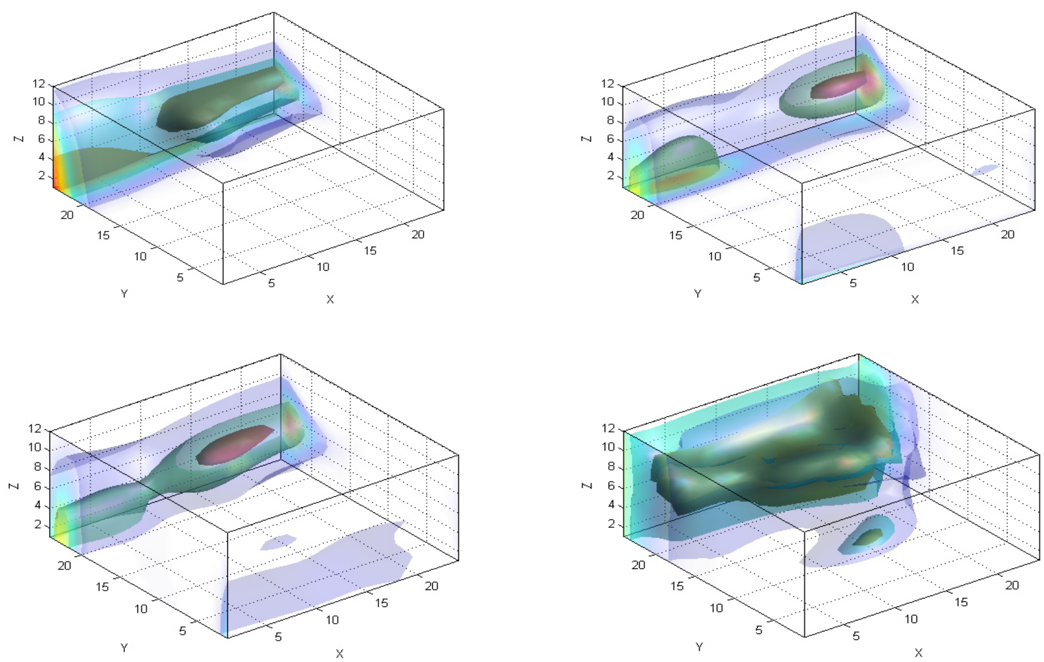

Figure 3. Upper left: the true initial seawater front shape ω_0; upper right: the estimate of ω_0 based solely on GW model; left bottom: the result of hydrogeophysical inversion for ω_0 ; right bottom: EM inversion only for ω_1. The level sets of solute fraction: ω = 0.1, 0.4 and 0.7 are displayed (red - 0.7 down to light blue - 0.1).

REFERENCES

D. Herckenrath, G. Fiandaca, E. Auken, P. Bauer-Gottwein 2013. Sequential and joint hydrogeophysical inversion using a field-scale groundwater model with ERT and TDEM data. Hydrology and Earth System Sciences, Discussions

P. Bauer-Gottwein, B. N. Gondwe, L. Christiansena, D. Herckenrath, L. Kgotlhangb, S. Zimmermann 2009. Hydrogeophysical exploration of three-dimensional salinity anomalies with the time-domain electromagnetic method (TDEM). Journal of Hydrology, Volume 380, Issue 3 – 4

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