A Laboratory Experiment of Fingertip Splitting During Variable-Density Flow

C. R. Guevara Morel, C. Cremer, N. Goldau and T. Graf
Institute of Fluid Mechanics and Environmental Physics in Civil Engineering, Leibniz Universität Hannover, Germany

ABSTRACT

The aim of this work is to understand plume fingertip splitting in homogeneous sand. A physical experiment is conducted in order to produce data that can be used to numerically simulate and test mathematical models and to determine which mathematical approximation (Oberbeck-Boussinesq approximation, OBA) adequately reproduces plume fingertip splitting. Numerically simulating fingering during variable-density flow is challenging because the appropriate OBA level is not yet known (Kolditz et al. 1998). The numerical simulations are realized with the HydroGeoSphere (HGS) model (Therrien et al. 2004). Fluid density changes in groundwater systems can occur among others due salt water intrusion in coastal aquifers, upconing of saline waters from deep aquifers and dense plumes coming from landfills. In these cases, a denser fluid overlies a less dense fluid producing a density gradient, forcing the denser fluid to move downward through the less dense fluid. For higher density contrasts, the migration of dense plumes typically results in the formation of instabilities which manifest in the form of vertical plume fingers. This physical process is relevant because it can accelerate vertical solute transport. Therefore, the understanding of fingering in variable-density flow processes is important for the adequate management of freshwater aquifers. In this contribution, we present results from a laboratory investigation where a denser fluid overlies a less dense fluid, and where the behavior of the plume was observed. Interestingly, one finger formed initially that underwent fingertip splitting to form 3 to 5 plume fingers. The total solute mass in the system and the plume penetration depth at selected times was recorded. In continuation of existing laboratory experiments for variable-density flow (e.g. Simmons et al. 2002), the current work contributes to the understanding of these fingertip splitting, including its generation and simulation. Results suggest that a higher mathematical approximation (OBA level 3) is needed to adequately predict variable-density fingering in case of high density contrasts.

Keywords: Density-driven flow, HydroGeoSphere, fingertip splitting, physical experiment, Oberbeck-Boussinesq approximation

INTRODUCTION

The importance of variable-density flow has been reported by Kolditz et al. (1998), and Diersch and Kolditz (2002) which reviewed fundamental concepts, state equations, and physical processes involved as well as benchmark problems and relevant studies in the field of variable-density flow. Recently, an extensive evaluation of advances in variable-density flow and transport is made by Simmons et al., (2010), in which physics, modelling approaches, benchmark problems as well as future challenges involved in the numerical modeling are discussed. When numerically simulating variable-density flow, different mathematical approximations (Oberbeck-Boussinesq approximations, OBA) are used (Oberbeck 1879, Boussinesq 1903; Kolditz et al. 1998; Oswald and Kinzelbach 2004) to accurately reproduce flow and transport. Normally, numerical models are verified by comparing results with analytical solutions but due to the nonlinear nature of variable-density flow problems, analytical solutions assume a sharp interface between a dense fluid overlying a light fluid (Bear and Dagan 1964) making the testing of a variable-density flow model´s ability to simulate fingering and convective mixing a major issue (Van Reeuwijk et al. 2009). Also, multiple steady-state solutions appear in variable-density benchmark problems (e.g. the solute analog Elder (1967) problem) depending on grid discretization making model verification a problem. Recently, Van Reeuwijk et al. (2009) found that a unique solution of the Elder problem can be obtained at a Rayleigh number of $Ra<76$. Van Reeuwijk et al. (2009) presented an analytical solution using a pseudospectral approach. The pseudospectral solution is discretization-independent and can therefore be used to verify variable-density flow models.

MATHEMATICAL MODEL

The HGS 3D numerical model describing fully-integrated variably-saturated subsurface and surface flow and variable-density solute transport was modified in order to be able to simulate the different mathematical OBA accuracies presented by Kolditz et al. (1998). The modified HGS model will then be used to reproduce the physical experiment.

LABORATORY MODEL

A laboratory experiment where a denser fluid overlies a less dense fluid is conducted. The experiment was carried out in a fully saturated sand filled glass container. A salt solution containing sodium chloride (NaCl) with a fluid density of 1200 kg m^{-3} (salt concentration of 348.61 g L^{-1} that is equivalent to 1000 % average seawater salinity) was used as the dense fluid. The fluid was stained with Eocene and introduced at the top boundary of the tank at a constant infiltration rate. Knowing the infiltration rate facilitates the calculation of the solute present in the system in time. Microscopic characteristic such as the number of fingers formed resulting from fingertip splitting as well as macroscopic characteristics such as the total solute mass in the system and the plumes penetration depth at various times were recorded.

NUMERICAL AND EXPERIMENTAL RESULTS

For model verification, the low Rayleigh number Elder problem (LREP) was simulated at different mathematical accuracies (OBA levels) using the newly modified HGS code. Numerical results were then compared with the pseudospectral solution of level OBA 1 at different times. Figure 1 shows results from the numerical simulation results of the half domain of the LREP at a $Ra = 60$ at a dimensionless time of $t = 1$ which is equivalent to 200 years in real time. The maximum fluid density in the domain is 1200 kg m^{-3}. The horizontal and vertical axes are dimensionless lengths scaled with respect to the original depth of the solute analog Elder problem. It is shown that the numerical solution depicted as red dotted line (OBA level 1) is identical to the pseudospectral solution of level OBA 1, such that the numerical model is successfully verified. Also, the green line (higher mathematical accuracy, OBA level 3) diverges from the black line (pseudospectral solution of OBA level 1), such that the more solute transport is predicted compared to the green line.

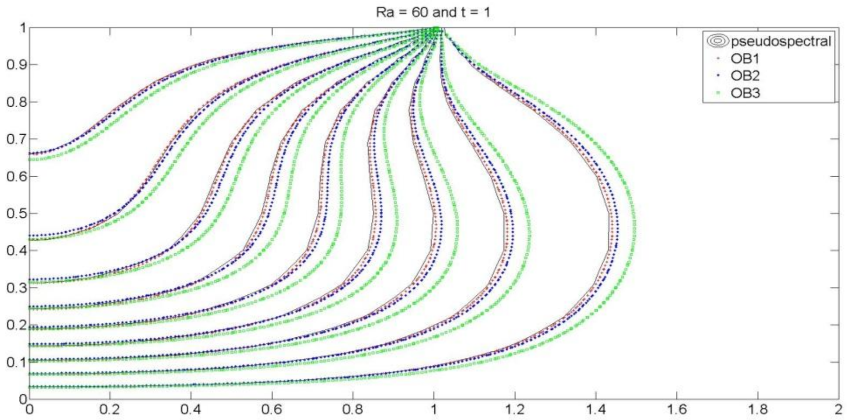

Figure 1. Results of model verification using the LREP . Black line: pseudospectral solution, red dotted line: numerical solution with lowr mathematical accuracy; blue dotted line: numerical solution with medium mathematical accuracy; green dotted line: numerical solution with high mathematical accuracy.

Figure 2 depicts results of the physical experiment. The source is located at the top boundary, and it is made sufficiently small in order to produce only one plume finger at early times. This is done in order try to understand fingertip splitting at later times. Solute mass present in the system, plume penetration depth and number of fingers is recorded.

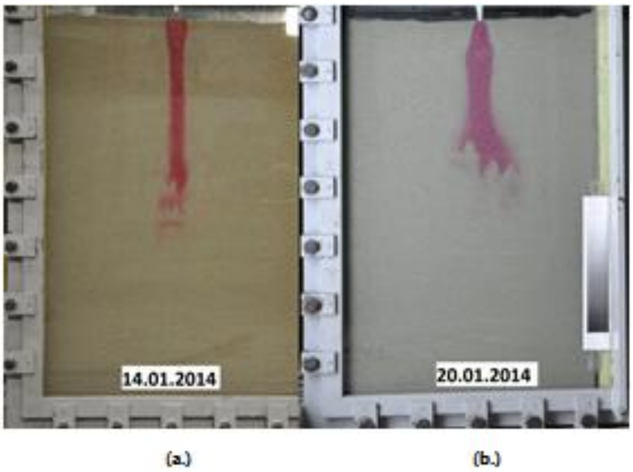

Figure 2. Results of the variable-density flow experiment at t = 15 min with an infiltration rate of 12.5 ml/min. The source is located (a) ca. 10 cm above the porous medium, and (b) within the porous medium

CONLUSIONS

The numerical experiments suggest that in the case of high density contrasts, a higher mathematical accuracy is needed to simulate variable-density flow. This finding may be an indicator that solute transport is underestimated when lower accuracies are used in case of the existence of high density gradients. Nevertheless the question of which mathematical reproduces nature more accurately is still not completely solved. The need of physical experiments in order to test numerical models and corroborate numerical results is evident. The developed model will be used to test appropriateness of the Oberbeck-Boussinesq assumptions in the simulation of fingertip splitting.

REFERENCES

Bear J., Dagan G. 1964. Some exact solutions of interface problems by means of the hodograph method. J. Geophys. R. 69: 1563-1572.

Boussinesq V.J. 1903. Theorie analytique de la chaleur, vol. 2. Paris, France: Gauthier-Villars; [chapter 2.3].

Diersch H.-J.G., Kolditz O. 2002. Variable-density flow and transport in porous media: approaches and challenges. Adv Water Resour 25(10): 899-944.

Elder J.W. 1967. Transient convection in a porous medium. J Fluid Mech 27(3): 609-23.

Kolditz O., Ratke R., Zielke W., Diersch H.-J.G. 1995. Coupled physical modelling for the analysis of groundwater systems. In Notes on Numerical Fluid Mechanics, Vol. 51, Vieweg, Braunschweig-Wiesbaden.

Oberbeck A. 1879. Ueber die Wärmeleitung der Flüssigkeiten bei Berücksichti-gung der Strömung infolge von Temperaturdifferenzen. Ann Phys Chem 7:271-92.

Oswald S.E., Kinzelbach W. 2004. Three-dimensional physical benchmark experiments to test variable-density flow models. J Hydrol 290(5):22-42.

Prasad A., Simmons C.T. 2005. Using quantitative indicators to evaluate results from variable-density groundwater flow models. Hydrogeol J 13(10):90514.

Simmons C.T., Bauer-Gottwein P., Graf T., Kinzelbach W., Kooi H., Li L., Post V., Prommer H., Therrien R., Voss C.I., Ward J., Werner A. 2010. Variable density groundwater flow: from modelling to applications. In Wheater H.S., Mathias S.A., Xin Li, eds. Groundwater Modelling in Arid and Semi-Arid Areas. 1st ed Cambridge: Cambridge University Press 7:87117.

Simmons C.T., Pierini M.L., Hutson J.L. 2002. Laboratory investigation of variable density flow and solute transport in unsaturated-saturated porous media. Transp Porous Media 47(2):21544.

Therrien R., McLaren R.G., Sudicky E.A., Panday S.M. 2004. Hydrogeosphere: a three-dimensional numerical model describing fully-integrated subsurface and surface flow and solute transport. Université Laval, University of Waterloo 275 pp.

van Reeuwijk M., Mathias S., Simmons C., Wards J. 2009. Insights from a Pseudospectral approach to the Elder problem. Water Resour Res 45,28 W04416, doi:10.1029/2008WR007421.

Contact Information: Carlos R. Guevara Morel, Leibniz Universität Hannover, Institute of Fluid Mechanics and Environmental Physics in Civil Engineering, Appelstrasse 9A 30167, Hannover, Germany, Phone: +49 511762 - 3710, Email: guevara@hydromech.uni-hannover.de