Assessment of Saltwater intrusion in the aquifer of Tripoli Lebanon

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ABSTRACT

The aquifer of Tripoli is located in the north of Lebanon and includes the city of Tripoli which is the second largest city with 600, 000 inhabitants. Fresh water supplies for Tripoli are provided from rivers surrounding the city and large pumping wells extracting water from the aquifer. In the last decade water demand increased drastically due to demographic development. Extensive pumping from municipality and personal wells has induced a decrease in the groundwater table and an aggravation of the saltwater intrusion phenomena. Measurements of water conductivity in selected wells have been done in 2008 and 2009 to assess the water quality.
In this study we develop a mathematical model for saltwater intrusion. The model is based on the 2D Boussinesq approximation considering two non-miscible fluids (fresh water and saltwater). We obtain a system of two equations with two unknowns. Considering the sharp interface approximation we reduce our system to one non-linear differential equation with one unknown: the fresh water head. We implemented the finite element method to solve the differential equation. The model is then applied to the Tripoli aquifer.

INTRODUCTION

The studied zone is a part of Tripoli that is on the Mediterranean Sea, its total area is $17.66\,km^2$, with Mediterranean climate whose temperature oscillates between $5^\circ C$ and $30^\circ C$. Like many coastal cities, Tripoli is impacted by the saltwater intrusion phenomena, this natural process is aggravated by the over - exploitation of fresh underground water. In this study we implement a 2D saltwater intrusion model based on the Boussineq equations coupled to the Ghyben - Herzberg approximation.

Background

Based on (Jazar & al., 2012) and (Bear,1979), we use mathematical transformation, perform numerical algorithm, then a tricky reconstruction that allow us to obtain numerical simulations of the two interfaces dry soil/ fresh water and fresh/ salter water of Tripoli basin.

METHODS

Geometric of the domain and boundary conditions

The figure 1 shows a schematic representation of a coastal aquifer subject to saltwater intrusion. Tripoli is divided into lower and upper zones. The altitude in the lower zone varies between 4 and 15 meters. We restrict our study on the lower zone of Tripoli. This zone rests on a horizontal substratum of Tripoli's aquifer at constant altitude 200 m below the sea level. The boundary of the studied zone is devised into two parts: Γ_1 which represents the shore and Γ_2 which is the boundary between the lower zone and the upper zone of Tripoli city.

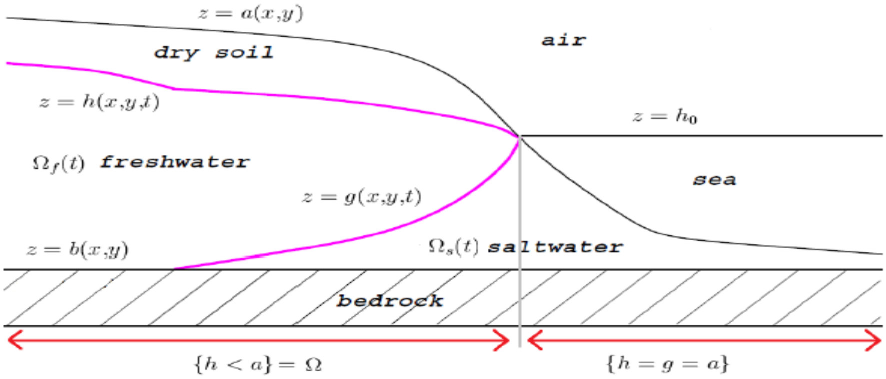

Figure 1: Schematic representation of the computational domain

Mathematical Model

We assume that we are in permanent regime. Indeed, the two interfaces z=h and z=g move very slowly. Moreover, we assume that b=cte since the substratum of the aquifer has a small variation. The two interfaces h and g are calculated using the following system (Jazar &.,al 2012) and (Bear, 1979):

$$\begin{cases} 0 = \operatorname{div}\left((h-g)\nabla(1-\varepsilon_0)h\right) & \text{in } \{h < a\} \\ 0 = \operatorname{div}\left((g-b)\nabla((1-\varepsilon_0)h+\varepsilon_0 g)\right) & \text{in } \{g<a\} \end{cases} \quad (1)$$

$\varepsilon_0 = \left(\rho_s - \rho_f\right)/\rho_s$ where ρ_s and ρ_f are the mass density of the salt and fresh water respectively. In permanent regime and considering a hydrostatic saltwater zone, the couple (h,g) satisfies the "Ghyben - Herzberg" relation (Bear, 1979):

$$\left(1-\varepsilon_0\right)h + \varepsilon_0 g = h_0 \quad (2)$$

Where h_0 is the sea level, assumed to be constant independent of time. The second equation of the system (1) is then trivial and the system (1) can be reduced to the first equation. Setting $\Omega_1 = \{g > b\}$ and $\Omega_2 = \{g = b\}$ then the function (3) satisfies

$$-\Delta u = 0 \quad \text{in } \Omega \quad (4)$$

where $\Omega = \Omega_1 \cup \Omega_2$ and

$$u = u(h) := \begin{cases} (h-b)^2 & \text{in } \{g=b\} \\ \left(\dfrac{h-h_0}{\varepsilon_0}\right)^2 & \text{in } \{g>b\} \end{cases} \quad (3)$$

Algorithm of the numerical implementation

The numerical computation of u as defined in (4) is easy. However, our concern is to determine the interface g, that is equivalent to the determination of the internal boundary between Ω_1 and Ω_2 . We can write:

$$\Omega_1 = \left\{ u < \left(\frac{h_0 - b}{1-\varepsilon_0}\right)^2 \right\} \text{ and } \Omega_2 = \left\{ u > \left(\frac{h_0 - b}{1-\varepsilon_0}\right)^2 \right\} \quad (5)$$

Indeed,

$$F(h) = (h-h_0)^2 - \varepsilon_0{}^2(h-h_0)^2 = (h(1-\varepsilon_0) - h_0 + \varepsilon_0 b)(h - h_0 + \varepsilon_0(h-b))$$

Then,

$\{F(h) < 0\} = \{h < h_1\}$ since in Ω_1, $u = \left(\frac{h-h_0}{\varepsilon_0}\right)^2$, then, $\Omega_1 = \left\{u < \left(\frac{h_1-h_0}{\varepsilon_0}\right)^2 = \left(\frac{h_0-b}{1-\varepsilon_0}\right)^2\right\}$,

where $h_1 = \frac{h_0 - \varepsilon_0 b}{1-\varepsilon_0}$. Similarly for Ω_2.

To summarize, the algorithm is as follows:

1. Numerical computation of u using equations (3) and (4).
2. Determination of the two domains Ω_1 and Ω_2. using equation (5).
3. Computation of the interface h as follows:

$$h = \begin{cases} h_0 + \varepsilon_0\sqrt{u} & \text{in } \Omega_1 \\ b + \sqrt{u} & \text{in } \Omega_2 \end{cases} \qquad (6)$$

4. Using Ghyben - Herzberg relation (2), we compute the interface g as follows:

$$g = \begin{cases} \frac{(h_0 - (1-\varepsilon_0)h)}{\varepsilon_0} & \text{in } \Omega_1 \\ b & \text{in } \Omega_2 \end{cases} \qquad (7)$$

RESULTS

The numerical simulations have been implemented in a rectangle of 6 km length and 2 km width by using the finite elements method. We use the software Freefem ++ to simulate the equation (4) (Hecht F., 2012). The substratum is considered as reference level: b = 0 m and $h_0 = 200$ m .Concerning the boundary conditions, we have $h = h_0$ on Γ_1 (the shore), on (Γ_2) we know that the interface h is at 7 m up to the level sea, then using (4) we have

$$u = \begin{cases} 0 & \text{on } \Gamma_1 \\ (207-0)^2 & \text{on } \Gamma_2 \end{cases}.$$

Finally, we have the following system:

$$\begin{cases} -\Delta u = 0 & \text{in } \Omega \\ u=0 & \text{on } \Gamma_1 \\ u=42849 & \text{on } \Gamma_2 \end{cases} \qquad (8).$$

The "FreeSWIM" method is validated using numerical software called "BFSWIM" that is a computational fluid dynamics for porous media flow and hydrological systems, taking into account 3D heterogeneity, anisotropy, and various couplings (Ababou R., 1993) and (Al - Bitar, A, and Ababou, R., 2005). The analytical solution introduced by (Bear, 1979) and Ghyben – Herzberg is also calculated in order to validate our method. Figure 2 shows that the interface g calculated by the "FreeSWIM" method is close to that calculated by the "BFSWIM" software and the analytical solution. Then, we implement this method on an aerial photo of the lower zone of Tripoli city, and the interface g is detected as shown in figure 3.

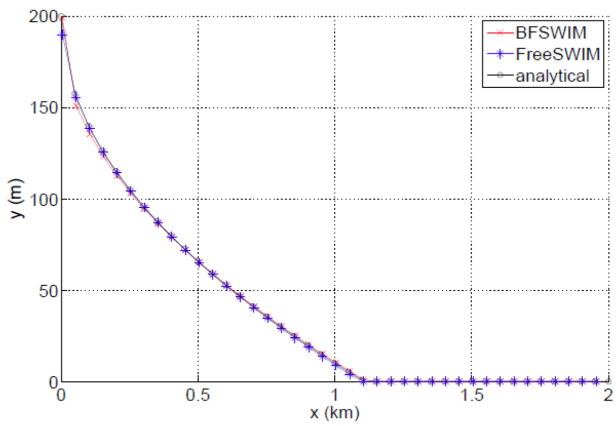

Figure 2: Comparison between analytic and numerical solutions

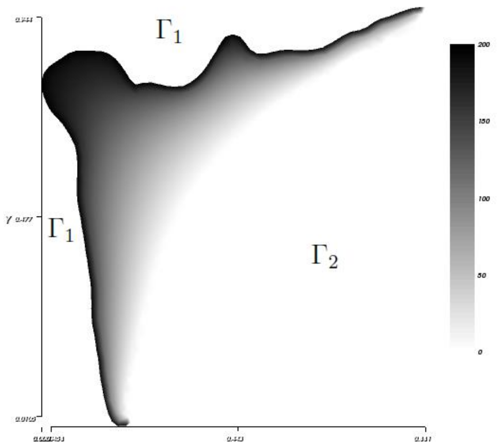

Figure 3: Freshwater/ saltwater interface in Tripoli

DISCUSSION AND CONCLUSIONS

To summarize, we introduced a new method that combines mathematical transformations, simple numerical computations and tricky computations that enable us to compute numerically the interface fresh / salt water. We qualitatively compare numerical simulations in steady state with the analytical solution and with the BFSWIM model, and then we implement the simulations on the lower zone of Tripoli city. Currently a model taking into consideration well pumping and transient state simulations is being implemented. More thorough and quantitative comparison with benchmark models and validation against wells measurements in the Tripoli aquifer will be done.

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